16 Multi-Photon Entanglement and Quantum Non-Locality

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We review recent experiments concerning multi-photon Greenberger–Horne–Zeilinger (GHZ) entanglement. We have experimentally demonstrated GHZ entanglement of up to four photons by making use of pulsed parametric down-conversion. On the basis of measurements on three-photon entanglement, we have realized the first experimental test of quantum non-locality following from the GHZ argument. Not only does multi-particle entanglement enable various fundamental tests of quantum mechanics versus local realism, but it also plays a crucial role in many quantum-communication and quantum-computation schemes.

16.1 Introduction

Ever since its introduction in 1935 by *Schrödinger* [1] entanglement has occupied a central position in the discussion of the non-locality of quantum mechanics. Originally the discussion focused on the proposal by *Einstein*, *Podolsky* and *Rosen* (EPR) of measurements performed on two spatially separated entangled particles [2]. Most significantly John Bell then showed that certain statistical correlations predicted by quantum physics for measurements on such two-particle systems cannot be understood within a realistic picture based on local properties of each individual particle – even if the two particles are separated by large distances [3].

An increasing number of experiments on entangled particle pairs having confirmed the statistical predictions of quantum mechanics [4–6] and have thus provided increasing evidence against local realistic theories. However, one might find some comfort in the fact that such a realistic and thus classical picture can explain perfect correlations and is only in conflict with statistical predictions of the theory. After all, quantum mechanics is statistical in its core structure. In other words, for entangled-particle pairs the cases where the result of a measurement on one particle can definitely be predicted on the basis of a measurement result on the other particle can be explained by a local realistic model. It is only that subset of statistical correlations where the measurement results on one particle can only be predicted with a certain probability which cannot be explained by such a model.

Surprisingly, in 1989 it was shown by *Greenberger*, *Horne* and *Zeilinger* (GHZ) that for certain three- and four-particle states [7, 8] a conflict with local realism arises even for perfect correlations. That is, even for those cases where, based on the measurement on $N-1$ of the particles, the result of the measurement on particle N can be predicted with certainty. Local realism and quantum mechanics here both make definite but completely opposite predictions.

The main purpose of this paper is to present a tutorial review on the recent progress concerning the first experimental realization of three- and four-photon GHZ entanglement [9, 10] and the first experimental test of GHZ theorem [11]. The paper is organized as follows: In Sect. 16.2 we briefly introduce the so-called GHZ theorem.In Sect. 16.3, we show in detail how pulsed parametric down-conversion can be used to generate multi-photon entanglement. In Sect. 16.4, we present the first three-particle test of local realism following from the GHZ theorem. Finally, the possible applications of the techniques developed in the experiments are briefly discussed in Sect. 16.5.

16.2 The GHZ Theorem

To show how the quantum predictions of GHZ states are in stronger conflict with local realism than the conflict for two-particle states as implied by Bell's inequalities, let us consider the following three-photon GHZ state:[1]

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2|H\rangle_3 + |V\rangle_1|V\rangle_2|V\rangle_3) , \tag{16.1}$$

where H and V denote horizontal and vertical linear polarizations, respectively. This state indicates that the three photons are in a quantum superposition of the state $|H\rangle_1|H\rangle_2|H\rangle_3$ (all three photons are horizontally polarized) and the state $|V\rangle_1|V\rangle_2|V\rangle_3$ (all three photons are vertically polarized), with none of the photons having a well-defined state of its own.

Consider now measurements of linear polarization along directions H'/V' rotated by 45° with respect to the original H/V directions, or of circular polarization, L/R (left-handed, right-handed). These new polarizations can be expressed in terms of the original ones as

$$|H'\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle) , \quad |V'\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle) , \tag{16.2}$$

$$|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + \mathrm{i}\,|V\rangle) , \quad |L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - \mathrm{i}\,|V\rangle) . \tag{16.3}$$

Let us denote $|H\rangle$ by matrix $\begin{pmatrix}1\\0\end{pmatrix}$ and $|V\rangle$ by matrix $\begin{pmatrix}0\\1\end{pmatrix}$; they are thus the two eigenstates of the Pauli operator, σ_z, correspondingly with the eigenvalues $+1$ and -1. We can also easily verify that $|H'\rangle$ and $|V'\rangle$ or $|R\rangle$ and $|L\rangle$ are

[1] The same line of reasoning can also be applied to the four-particle case.

two eigenstates for the Pauli operator σ_x or σ_y with the values $+1$ and -1, respectively. For convenience we will refer to a measurement of the H'/V' linear polarization as an x measurement and of the L/R circular polarization as a y measurement.

By representing state (16.1) in the new states using (16.2) and (16.3), one obtains the quantum predictions for the measurements of these new polarizations. For example, for the case of the measurement of circular polarization on, say, both photons 1 and 2 and the measurement of linear polarization H'/V' on photon 3, denoted as a yyx experiment, the state may be expressed as

$$|\Psi\rangle = \frac{1}{2}(|R\rangle_1 |L\rangle_2 |H'\rangle_3 + |L\rangle_1 |R\rangle_2 |H'\rangle_3 + |R\rangle_1 |R\rangle_2 |V'\rangle_3 + |L\rangle_1 |L\rangle_2 |V'\rangle_3) \,. \tag{16.4}$$

This expression implies, first, that any specific result obtained in any individual or in any two-photon joint measurement is maximally random. For example, photon 1 will exhibit polarization R or L with the same probability of 50%, or photons 1 and 2 will exhibit polarizations RL, LR, RR, or LL with the same probability of 25%. Second, given any two results of measurements on any two photons, we can predict with certainty the result of the corresponding measurement performed on the third photon. For example, suppose photons 1 and 2 both exhibit right-handed (R) circular polarization. By the third term in (16.4), photon 3 will definitely be V' polarized.

By cyclic permutation, we can obtain analogous expressions for any experiment measuring circular polarization on two photons and H'/V' linear polarization on the remaining one. Thus, in every one of the three yyx, yxy and xyy experiments, any individual measurement result – both for circular polarization and for linear H'/V' polarization – can be predicted with certainty for every photon given the corresponding measurement results of the other two.

We now analyze the implications of these predictions from the point of view of local realism. First, note that the predictions are independent of the spatial separation of the photons and independent of the relative time order of the measurements. Let us thus consider the experiment to be performed such that the three measurements are performed simultaneously in a given reference frame, say, for conceptual simplicity, in the reference frame of the source. Thus we can employ the notion of Einstein locality, which implies that no information can travel faster than the speed of light. Hence the specific measurement result obtained for any photon must not depend on which specific measurement is performed simultaneously on the other two or on the outcome of these measurements. The only way then to explain from a local realistic point of view the perfect correlations discussed above is to assume that each photon carries elements of reality for both x and y measurements considered and that these elements of reality determine the specific individual measurement result [7, 8, 12].

Calling these elements of reality, of photon i, X_i with values $+1(-1)$ for $H'(V')$ polarizations and Y_i with values $+1(-1)$ for $R(L)$ polarizations we obtain the relations $Y_1Y_2X_3 = -1$, $Y_1X_2Y_3 = -1$ and $X_1Y_2Y_3$ in order to be able to reproduce the quantum predictions of (16.4) and its permutations [12].

We now consider a fourth experiment measuring linear H'/V' polarization on all three photons, that is, an xxx experiment. We investigate the possible outcomes that will be predicted by local realism based on the elements of reality introduced to explain the earlier yyx, yxy and xyy experiments.

Because of Einstein locality any specific measurement for x must be independent of whether an x or y measurement is performed on the other photon. As $Y_iY_i = +1$, we can write $X_1X_2X_3 = (X_1Y_2Y_3) \cdot (Y_1X_2Y_3) \cdot (Y_1Y_2X_3)$ and obtain $X_1X_2X_3 = -1$. Thus from a local realistic point of view the only possible results for an xxx experiment are $V'_1V'_2V'_3$, $H'_1H'_2V'_3$, $H'_1V'_2H'_3$, and $V'_1H'_2H'_3$.

How do these predictions of local realism for an xxx experiment compare with those of quantum physics? If we express the state given in (16.1) in terms of H'/V' polarization using (16.2), we obtain

$$|\Psi\rangle = \frac{1}{2}(|H'\rangle_1 |H'\rangle_2 |H'\rangle_3 + |H'\rangle_1 |V'\rangle_2 |V'\rangle_3 + |V'\rangle_1 |H'\rangle_2 |V'\rangle_3 + |V'\rangle_1 |V'\rangle_2 |H'\rangle_3) \,. \quad (16.5)$$

We conclude that the local realistic model predicts none of the terms occurring in the quantum prediction and vice versa. This implies that, whenever local realism predicts a specific result definitely to occur for a measurement on one of the photons based on the results for the other two, quantum physics definitely predicts the opposite result. For example, if two photons are both found to be H' polarized, local realism predicts the third photon to carry V' polarization while the quantum state predicts H' polarization. This is the GHZ contradiction between local realism and quantum physics.

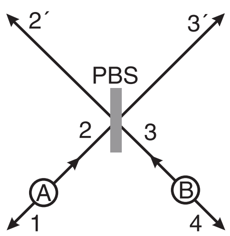

Fig. 16.1. Principle for observing three- or four-photon GHZ correlations. Sources A and B each deliver one entangled particle pair. A polarizing beam-splitter (PBS) combines modes 2 and 3. The two photons detected in its output port are either both H (horizontally) or both V (vertically) polarized, projecting the complete four-photon state into a GHZ state

In the case of Bell's inequalities for two photons the conflict between local realism and quantum physics arises for statistical predictions of the theory; but for three entangled particles the conflict arises even for the definite predictions. Statistics now only results from the inevitable experimental limitations occurring in any and every experiment, even in classical physics.

16.3 Experimental Multi-Photon GHZ Entanglement

Experimental testing of the GHZ theorem necessitates observations of multi-particle entanglement. The method used here to generate multi-photon GHZ entanglement is a further development of the techniques that have been used in our previous experiments on quantum teleportation [13] and entanglement swapping [14]. The main idea, as was put forward in [15], is to transform two independently created photon pairs into either three- or four-photon entanglement. The working principle is shown in Fig. 16.1.

Suppose that the two pairs are in the state

$$\begin{aligned}\left|\Psi^i\right\rangle_{1234} = &\frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2\right) \\ &\otimes \frac{1}{\sqrt{2}}\left(|H\rangle_3 |V\rangle_4 - |V\rangle_3 |H\rangle_4\right) ,\end{aligned} \tag{16.6}$$

which is a tensor product of two polarization entangled photon pairs.

One photon out of each pair is directed to the two inputs of a polarizing beam-splitter (PBS). Since the PBS transmits horizontal and reflects vertical polarization, coincidence detection between the two PBS outputs implies that photons 2 and 3 are either both horizontally polarized or both vertically polarized, and thus projects (16.6) onto a two-dimensional subspace spanned by $|V\rangle_1|H\rangle_2|H\rangle_3|V\rangle_4$ and $|H\rangle_1|V\rangle_2|V\rangle_3|H\rangle_4$.

After the PBS, the renormalized state corresponding to a four-fold coincidence is

$$\left|\Psi^f\right\rangle_{12'3'4} = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_{2'} |V\rangle_{3'} |H\rangle_4 + |V\rangle_1 |H\rangle_{2'} |H\rangle_{3'} |V\rangle_4\right) , \tag{16.7}$$

which is a GHZ state of four particles.

The scheme described above does not only yield four-particle entanglement but – assuming perfect pair sources and single-photon detectors – could also produce freely propagating three-particle entangled states via so-called entangled entanglement [16]. For example, one could analyze the polarization state of photon $2'$ by passing it through a special PBS that transmits H' polarizations but reflects V' ones. Detecting one photon in one of the two outputs of this PBS makes sure that there will be exactly one photon in each of the outputs 1, $3'$, and 4. Correspondingly, the polarization state of the remaining three photons in modes 1, $3'$, and 4 will then be projected onto either the state

$$|\Psi\rangle_{13'4} = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |V\rangle_{3'} |H\rangle_4 + |V\rangle_1 |H\rangle_{3'} |V\rangle_4 \right) , \qquad (16.8)$$

if one detects an H' polarized photon in mode $2'$, or the state

$$|\Psi\rangle_{13'4} = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |V\rangle_{3'} |H\rangle_4 - |V\rangle_1 |H\rangle_{3'} |V\rangle_4 \right) , \qquad (16.9)$$

if one detects a V' polarized photon in mode $2'$.

Note that, due to the absence of perfect pair sources and perfect single-photon detectors, in our experiments both three- and four-photon entanglements [9, 10] are observed only under the condition that there is one and only one photon in each of the four outputs. As there are other detection events where, for example, two photons appear in the same output port, this condition might raise doubts about whether such a source can be used to test local realism. The same question arose earlier for certain experiments involving photon pairs [17, 18], where a violation of Bell's inequality was only achieved under the condition that both detectors used register a photon. It was often believed [19, 20] that such experiments could never, not even in their idealized versions, be genuine tests of local realism. However, this has been disproved [21]. Following the same line of reasoning, it has been recently shown [22] that our procedure permits a valid GHZ test of local realism. In essence, both the Bell and the GHZ arguments exhibit a conflict between detection events and the ideas of local realism.

We now describe our experimental verification of multi-photon entanglement. Since the methods used in our three- and four-photon experiments are basically the same, in the following we will only present the experimental results on the observation of four-photon entanglement. For details of our three-photon experiment, please see [9].

In our experiment (Fig. 16.2) we create polarization-entangled photon pairs by spontaneous parametric down-conversion from an ultraviolet femtosecond pulsed laser (~ 200 fs, $\lambda \simeq 394.5$ nm) in a β-BaB_3O_6(BBO) crystal [23]. The laser passes the crystal a second time having been reflected off a translatable mirror. In the reverse pass another conversion process may happen, producing an second entangled pair. One particle of each pair is steered to a polarizing beam-splitter, where the path lengths of each particle have been adjusted such that they arrive simultaneously. On the polarizing beam-splitter a horizontally polarized photon will always be transmitted, whereas a vertically polarized one will always be reflected, both with less than a 10^{-3} error rate. The two outputs of the polarizing beam-splitter are spectrally filtered (3.5 nm bandwidth) and monitored by fiber-coupled single-photon counters (D2 and D3). The filtering process stretches the coherent time to about 550 fs, substantially larger than the pump-pulse duration [24]. This

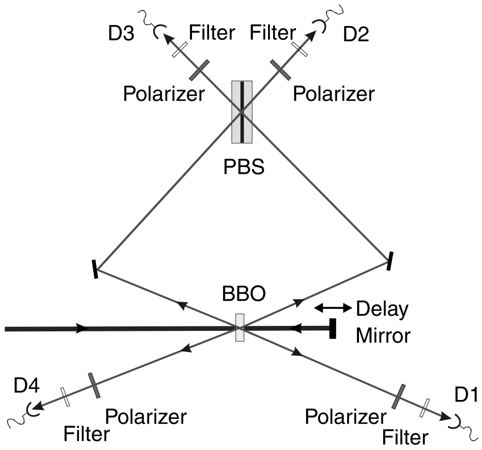

Fig. 16.2. Schematic of the experimental setup for the measurement of four-photon GHZ correlations. A pulse of UV light passes a BBO crystal twice to produce two entangled photon pairs. Coincidences between all four detectors, D1–4, exhibit GHZ entanglement

effectively erases any possibility of distinguishing the two photons according to their arrival time and therefore leads to interference.

The remaining two photons – one from each pair – pass identical filters in front of detectors D1 and D4 and are detected directly afterwards. In front of each of the four detectors we may insert a polarizer to assess the correlations with respect to various combinations of polarizer orientations. A correlation circuit extracts only those events where all four detectors registered a photon within a small time window of a few ns. This is necessary in order to exclude cases in which only one pair is created or two pairs in one pass of the pump pulse and none in the other.

To experimentally demonstrate that the state $|\Psi^f\rangle$ of (16.7) has been obtained, we first verified that under the condition of having a four-fold coincidence only the $HVVH$ and $VHHV$ components can be observed, but no others. This was done by comparing the count rates of all 16 possible polarization combinations, $HHHH \ldots VVVV$. The measurement results in the H/V basis (Fig. 16.3) show that the signal-to-noise ratio defined as the ratio of any of the desired four-fold events ($HVVH$ and $VHHV$) to any of the 14 other non-desired ones is about 200:1.

Showing the existence of $HVVH$ and $VHHV$ terms alone is a necessary but not sufficient experimental criterion for the verification of the state $|\Psi^f\rangle$, since the above observation is, in principle, compliant both with $|\Psi^f\rangle$ and with a statistical mixture of $HVVH$ and $VHHV$. Thus, as a further test we have to demonstrate that the two terms $HVVH$ and $VHHV$ are indeed in coherent superposition.

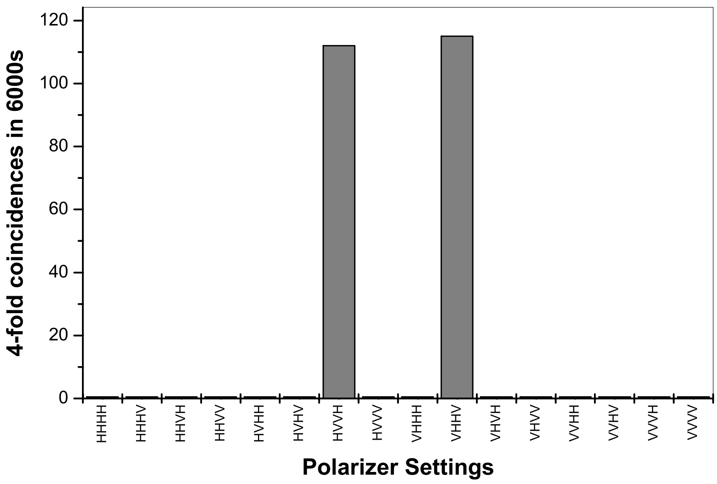

Fig. 16.3. Experimental data for horizontal and vertical polarizer settings. Only the two desired terms are present; all other terms which are not part of the state $|\Psi^f\rangle$ (16.7) are so strongly suppressed that they can hardly be discerned in the graph. The number of four-fold coincidences for any of the non-desired terms is 0.5 in 6000 s on average, i.e., seven events for all 14 possibilities

This was done by performing a polarization measurement in the H'/V' basis. Transforming $|\Psi^f\rangle$ to the H'/V' linear polarization basis yields an expression containing eight (out of 16 possible) terms, each with an even number of $|H'\rangle$ components. Combinations with odd numbers of $|H'\rangle$ components do not occur. As a test for coherence we can now check the presence or absence of various components. In Fig. 16.4 we compare the $(H'/H'/H'/H')$ and $(H'/H'/H'/V')$ count rates as a function of the pump delay mirror position. At zero delay – photons 2 and 3 arrive at the PBS simultaneously – the latter component is suppressed with a visibility of 0.79 ± 0.06. As explained in [24], many efforts have been made by us to obtain this high visibility reliably. In the experiment we observed that the most important ingredients for a high interference contrast were a high single pair entanglement quality, the use of narrow bandwidth filters, and the high quality of the polarizing beam-splitter.

These measurements clearly show that we obtained four-particle GHZ correlations. The quality of the correlations can be judged by the density matrix of the state

$$\rho = 0.89 \left|\Psi^f\right\rangle \left\langle\Psi^f\right|_{12'3'4} + 0.11 \left|\Phi\right\rangle \left\langle\Phi\right|_{12'3'4} , \tag{16.10}$$

where $|\Phi\rangle = 1/\sqrt{2}(|HVVH\rangle - |VHHV\rangle)$. This density matrix describes our data under the experimentally well-justified assumption that only phase errors in the H/V basis are present, which appear as bit-flip errors in the H'/V' basis (see Fig. 16.4).

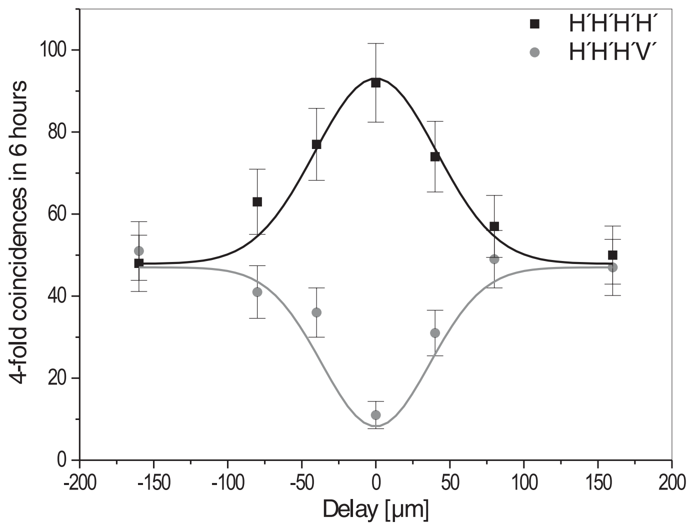

Fig. 16.4. Experimental data for 45° polarizer settings. The difference between the four-fold coincidence count rates for $(H'/H'/H'/H')$ and $(H'/H'/H'/V')$ shows that the amplitudes depicted in Fig. 16.3 are in coherent superposition. Maximum interference occurs at zero delay between photons 2 and 3 arriving at the polarizing beam-splitter. The Gaussian curves that roughly connect the data points are only shown to guide the eye. Visibility and errors are calculated only from the raw data

Since performing a polarization decomposition in the H'/V' basis in outputs $2'$ and $3'$ and a subsequent coincidence detection [25] serves exactly the role of Bell-state measurement, we emphasize that our four-photon experiment above can also be viewed as a high-fidelity realization of entanglement swapping, or equivalently teleportation of entanglement. Specifically, the data of Fig. 16.4 indicate that the state of, say, photon 2 was teleported to photon 4 with a fidelity of 0.89. This clearly outperforms our earlier work [14] in this field, and for the first time fully demonstrates the non-local feature of quantum teleportation [26].

16.4 Experimental Test of Quantum Non-Locality

Utilizing our source developed for three-photon GHZ entanglement [9], let us now present the first three-particle test of quantum nonlocality [11]. As explained in Sect. 16.2, demonstration of the conflict between local realism and quantum mechanics for three-photon GHZ entanglement consists of four experiments, each with three spatially separated polarization measurements.

First, one performs yyx, yxy, and xyy experiments. If the results obtained are in agreement with the predictions for a GHZ state, then the predictions for an xxx experiment for a local realist theory are exactly opposite to those for quantum mechanics.

For each experiment we have eight possible outcomes of which ideally four should never occur. Obviously, no experiment either in classical physics or in quantum mechanics can ever be perfect, and therefore, due to principally unavoidable experimental errors, even the outcomes which should not occur will occur with some small probability in any realistic experiment.

All individual fractions which were obtained in our yyx, yxy and xyy experiments are shown in Fig. 16.5a–c, respectively. From the data we conclude that we observe the GHZ terms of (16.4) predicted by quantum mechanics in 85% of all cases, and in 15% we observe spurious events.

If we assume the spurious events are just due to experimental errors, we can thus conclude within the experimental accuracy that for each photon, 1, 2 and 3, quantities corresponding to both x and y measurements are elements of reality. Consequently, a local realist, if he accepts that reasoning, would thus predict that for a xxx experiment only the combinations $V'V'V', H'H'V', H'V'H'$, and $V'H'H'$ will be observable (Fig. 16.6b). However, referring back to our original discussion, we see that quantum mechanics predicts that the exact opposite terms should be observed (Fig. 16.6a). To settle this conflict we then perform the actual xxx experiment. Our results, shown in Fig. 16.6c, disagree with the local realism predictions and are consistent with the quantum-mechanical predictions. The individual fractions in Fig. 16.6c clearly show within our experimental uncertainty that only those triple coincidences predicted by quantum mechanics occur and not those predicted by local realism. In this sense, we claim that we have experimentally realized the first three-particle test of local realism following the GHZ argument.

We have already seen that the observed results for an xxx experiment confirm the quantum-mechanical predictions when we assume that deviations from perfect correlations in our experiment, and in any experiment for that matter, are just due to unavoidable experimental errors. However, a local realist might argue against that approach and suggest that the non-perfect detection events indicate that the original GHZ argumentation cannot succeed.

To address this argument, a number of inequalities for N-particle GHZ states have been derived [27–29]. For instance, Mermin's inequality for a three-particle GHZ state reads as follows [27]:

$$|\langle\sigma_x\sigma_y\sigma_y\rangle + \langle\sigma_y\sigma_x\sigma_y\rangle + \langle\sigma_y\sigma_y\sigma_x\rangle - \langle\sigma_x\sigma_x\sigma_x\rangle| \leq 2 \; , \tag{16.11}$$

where symbol $\langle\cdot\rangle$ denotes the expectation value of a specific physical quantity. The necessary visibility to violate this inequality is 50%. The visibility observed in our GHZ experiment is $71 \pm 4\%$ and obviously surpasses the 50%

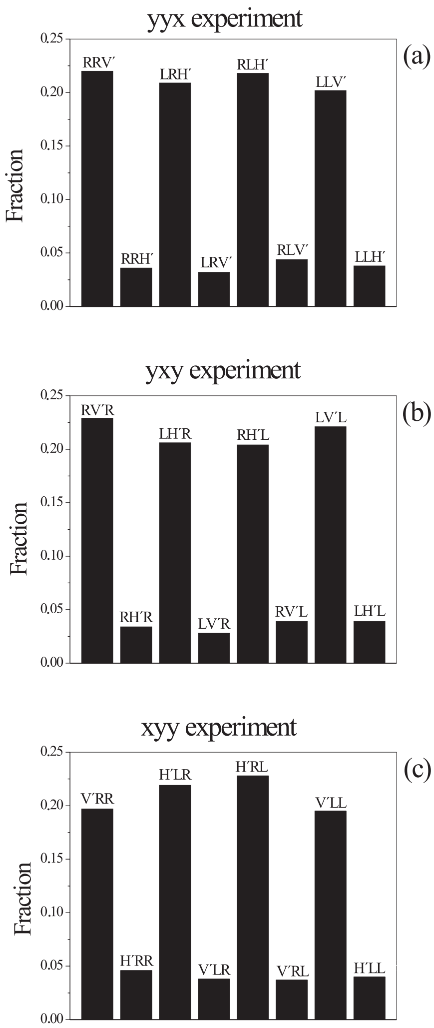

Fig. 16.5. Fractions of the various outcomes observed in the (**a**) yyx, (**b**) yxy, and (**c**) xyy experiments. The experimental data show that we observe the GHZ terms predicted by quantum physics in 85% of all cases and the spurious events in 15%

limitation. Substituting our results measured in the yyx, yxy and xyy experiments into the left-hand side of (16.11), we obtain the following constraint:

$$\langle \sigma_x \sigma_x \sigma_x \rangle \leq -0.1 \; , \tag{16.12}$$

by which a local realist can thus predict that in an xxx experiment the probability fraction for the outcomes yielding a $+1$ product, denoted by $P(xxx = +1)$, should be no larger than 0.45 ± 0.03 (also refer to the first bar in Fig. 16.7).

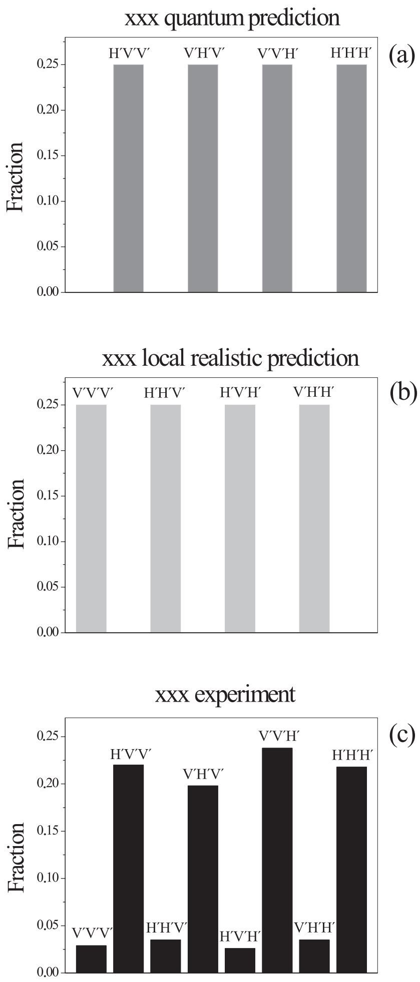

Fig. 16.6. The conflicting predictions of (**a**) quantum physics and (**b**) local realism of the fractions of the various outcomes in a xxx experiment for perfect correlations. (**c**) The experimental results are in agreement with quantum physics within experimental errors and in disagreement with local realism

What is the quantum prediction for an xxx experiment following from the yyx, yxy and xyy experimental results? Because our experimental visibility is due mainly to the finite width of the interference filters and the finite pulse duration, quantum mechanically it is expected that the same visibility should be observed in an xxx experiment; hence we obtain the quantum prediction as shown in the second bar of Fig. 16.7.

The visibility observed in our xxx experiment is $74 \pm 4\%$, which consequently gives $P(xxx = +1) = 0.87 \pm 0.04$ (shown in the third bar of

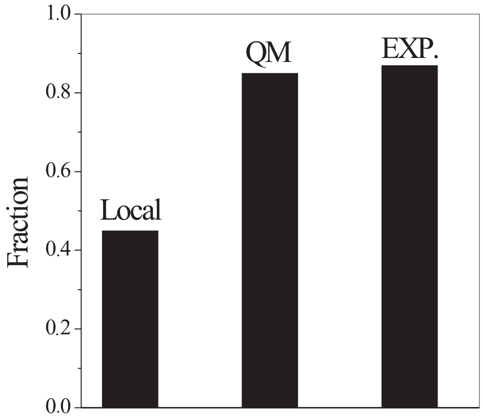

Fig. 16.7. Predictions of local realism (Local) and quantum physics (QM) for the probability fraction of the outcomes yielding a +1 product in an xxx experiment based on the real experimental data measured in the yyx, yxy and xyy experiments. The experimental results (EXP.) are in good agreement with quantum physics and in distinct conflict with local realism

Fig. 16.7). Comparing the results in Fig. 16.7, we therefore conclude that our experimental results verify the quantum prediction while they contradict the local-realism prediction by over 8 standard deviations; there is no local hidden-variable model which is capable of describing our experimental results.

16.5 Discussions and Prospects

The experimental realization of multi-particle GHZ entanglement and high-fidelity teleportation has rather profound implications. First, in higher entangled systems the contradiction with local realism becomes ever stronger, because both the necessary visibility and the required number of statistical tests to reject the local hidden-variable models at a certain confidence level decrease with the number of particles that are entangled [29, 30]. Second, based on the observed visibility of 0.79 ± 0.06, one could violate – with an appropriate set of polarization correlation measurements – Bell's inequality for photons 1 and 4, even though these two photons never interact directly. As noted by *Aspect*, "This would certainly help us to further understand nonlocality" [31].

Besides its significance in tests of quantum mechanics versus local realism, the methods developed in the experiment also have many useful applications in the field of quantum information. It was noticed very recently that, while our four-photon setup directly provides a simple way to perform entanglement concentration [32–34], a slight modification of the setup also provides a novel way to perform entanglement purification for general mixed entangled states [10]. Furthermore, following the recent proposal by *Knill* et al. [35], our multi-photon experiment also opens the possibility to experimentally investigate the basic elements of quantum computation with linear optics.

In summary, we have demonstrated a method of creating higher-order entangled states which can, in principle, be extended to any desired number of particles, provided one has efficient pair sources. Given that, more photon-pair sources could be combined with polarizing beam-splitters to yield entangled states of arbitrary numbers of particles. The latest developments in photon-pair sources suggest that it should be possible in the near future to have sources with many orders of magnitude higher emission rates [36]. With these entanglement sources one would be able to implement some quantum-computation algorithms using only entanglement and linear optics [35]. Also, more elaborate entanglement-purification protocols and high-fidelity teleportation over multiple stages as required for the construction of quantum repeaters [37] become possible.

Acknowledgement

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