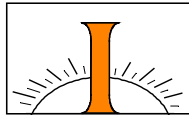


# **VEDIC MATH GENIUS**

**“What The Ancients Knew That  
Can Help Anyone Achieve Better  
Grades In The Math Class”**

**BY KENNETH WILLIAMS**



**INSPIRATION BOOKS  
2003**

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## FOREWORD

This book introduces the unique and extraordinary system of Vedic Mathematics, a system that has a unity and simplicity that is mostly missing in our modern 'system'.

If you have seen the book "Fun with Figures" you will already be familiar with some neat methods of calculation that are really striking. But these are only a part of a much larger system, indeed the Vedic system covers all of mathematics, both pure and applied.

In this book you will see some of the techniques in "Fun with Figures" extended and developed, and also many things not shown in that book. Here you will see illustrated all of the sixteen Sutras on which the system is based. You will need to study the material carefully but once you see the idea being introduced at each step you will find that the methods are actually very easy and quite natural.

- The Vedic system is simple and direct – you get the answer usually in one line.
- The Vedic system is flexible – you will see how to calculate from left to right as well as from right to left, for example.
- The system gives a choice of methods, so that you don't have to rigidly apply the one 'correct' method. This leads to increased creativity and adds to the fun of doing math.

This is therefore all of immense practical value, quite apart from increasing your mental agility and confidence, improving your memory and enabling you to do quick and efficient calculations when you need to.

By the time you finish the tenth chapter you should feel you have a good grasp of this beautiful system of Vedic Mathematics: "Mathematics with Smiles".

# INTRODUCTION

About a hundred years ago the scholars in the west discovered the Indian Vedas: ancient texts in their millions containing some of the most profound knowledge. In fact the Sanskrit word 'Veda' means 'knowledge'. They found Vedic texts on medicine, architecture, astronomy, ethics etc. etc. and according to the Indian tradition all knowledge is contained in these Vedas.

The age of these Vedic texts is disputed, tradition telling us that originally the Vedas were carried on by word of mouth, by pandits who recited the texts exactly and passed them on to their children.

However certain writings that were entitled Ganita Sutras, which means mathematics, were not found to contain any mathematics and the scholars rejected them saying they were nonsense.

Now a brilliant eastern scholar, Sri Bharata Krsna Tirthaji (1884-1960), heard what was being said about the Ganita texts and determined to uncover the mathematics in them. Between 1911 and 1918 he studied the texts and eventually was able to reconstruct the system that had been lost long ago.

He discovered that all of mathematics is governed by just sixteen sutras or word-formula (and some sub-Sutras), and he showed some really amazing examples of their power and versatility. He wrote sixteen books (one on each sutra) but these have been lost. In 1958 when the loss was confirmed he wrote an introductory book, which is currently available (Reference 1).

Recent attempts to reconstruct the system from this book have led to an upsurge of interest and the Vedic system which being taken up by researchers, teachers and others all over the world.

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# 1 GIVE AND TAKE

---

**A** Addition and subtraction always seem to come together. For example, if you take some money out of your pocket it is now in your hand but is no longer in your pocket.

So it is no surprise that there is a Vedic Sutra for this:

*By Addition and By Subtraction*

- \* We naturally use this when, say, we **add 55 and 19**.  
Most people will add 20 and take 1 away, so  $55 + 19 = 74$ .
- \* Similarly for  $198 + 64$  we could add 200 to 64 and take off 2 after.  
So  **$198 + 64 = 262$** .



Try these:

**a**  $39 + 44$

**b**  $33 + 38$

**c**  $48 + 35$

**d**  $27 + 34$

**e**  $33 + 198$

**f**  $297 + 167$


**g**  $18 + 19$

**h**  $29 + 38$

In the same way we can subtract easily:

- \* For  $44 - 19$  we take 20 from 44 and add 1 back on.  
So  **$44 - 19 = 25$** .
- \* And for  $333 - 198$  we can take 200 from 333 and add 2 back.  
We get  **$333 - 198 = 135$** .

*“From the intrinsic evidence of his creation, the Great Architect of the Universe now begins to appear as a pure mathematician.”  
Sir James Jeans*

 a $44 - 19$	b $66 - 29$	c $288 - 49$	d $155 - 28$
e $555 - 199$	f $881 - 57$	g $474 - 198$	h $777 - 388$

This is what many people would naturally do.

**B** If you have seen the Fun with Figures book you will have seen the really easy way to subtract from a **base number** like 10, 100, 1000 etc.

For this we use the Sutra:

*All from 9 and the Last from 10*

\* To **take 357 from 1000** for example,  
we apply *All from 9 and the Last from 10* to the figures of 357.

So  **$1000 - 357 = 643$** .

We take the 3 from 9 to get 6,  
take the 5 from 9 to get 4,  
and take the 7 from 10 to get 3.

\* Similarly  **$10,000 - 7621 = 2379$** ,  
we take 7, 6 and 2 from 9 and the 1 from 10.

 Try these:

i $1000 - 246$	j $1000 - 817$	k $10,000 - 3489$	l $100 - 28$
----------------	----------------	-------------------	--------------

If the number of figures in the number being subtracted is less than the number of zeros in the base number, we just add as many zeros as needed to make them the same:

\* For example in  **$10,000 - 643$**  there are four zeros and only three figures in 643.

So we write  $10,000 - 0643$  so that 0643 has four figures.  
Then  $10,000 - 0643 = 9357$ .  
(We take 0, 6, 4 from 9 and 3 from 10)

\* Similarly,  $10,000 - 77 = 10,000 - 0077 = 9923$ .



**a**  $1000 - 46$

**b**  $10,000 - 727$

**c**  $10,000 - 84$

**d**  $10,000 - 208$

Now suppose we don't have a base number, but a multiple of it: like 200, 3000 etc.

\* Suppose you had  $3000 - 444$ .  
Can you see what to do?

We get  $3000 - 444 = 2556$ .

The 444 will come off one of the three thousands, so the initial 3 becomes 2.

Then applying "All from 9 and the Last from 10" to the 444 we put down 556.

\* Similarly  $700 - 52 = 648$ .  
The 7 reduces to 6, and applying the sutra to 52 gives 48.



Try these:

**e**  $4000 - 246$

**f**  $700 - 38$

**g**  $50,000 - 7489$

**h**  $9000 - 802$

There is one thing to be careful of though.

\* Suppose you had  $7000 - 430$ .  
When you apply the Sutra to 430 you just apply it to the 43, not to 430.

So you get  $7000 - 430 = 6570$ .  
(7 becomes 6, the Sutra converts 43 to 57 and the zero is carried over)



**C** Amazingly, this simple method can handle all subtractions!

\* Suppose we had:  $444 - 286$ .

We can set the sum out like this:

$$\begin{array}{r} 444 \\ \underline{286} - \\ \hline \end{array}$$

**Subtracting in each column** we get  $4 - 2 = 2$ ,  $4 - 8 = -4$ ,  $4 - 6 = -2$ .

Since these negative answers can be written with **the minus on top** we can write:

$$\begin{array}{r} 444 \\ \underline{286} - \\ \underline{242} \end{array}$$

and  $2\bar{4}\bar{2}$  is easily converted into 158 because this is just the kind of sum you have just done:  
 $2\bar{4}\bar{2}$  means  $200 - 42 = 158$ .

 Try these:

**a**  $\begin{array}{r} 543 \\ \underline{168} - \\ \hline \end{array}$

**b**  $\begin{array}{r} 567 \\ \underline{279} - \\ \hline \end{array}$

**c**  $\begin{array}{r} 804 \\ \underline{388} - \\ \hline \end{array}$

**d**  $\begin{array}{r} 7347 \\ \underline{5589} - \\ \hline \end{array}$

\* Similarly

$$\begin{array}{r} 6767 \\ \underline{1908} - \\ \underline{5\bar{2}6\bar{1}} = 4859. \end{array}$$

Here, to convert  $5\bar{2}6\bar{1}$  to 4859 we split it into  $5\bar{2}/6\bar{1}$ .

$5\bar{2} = 48$  (because  $50 - 2 = 48$ ), and  $6\bar{1} = 59$  (because  $60 - 1 = 59$ ).

**e**  $\begin{array}{r} 8074 \\ \underline{5339} - \\ \hline \end{array}$

**f**  $\begin{array}{r} 6593 \\ \underline{2877} - \\ \hline \end{array}$

**g**  $\begin{array}{r} 7162 \\ \underline{3916} - \\ \hline \end{array}$

## 2

# MIGHTY MULTIPLICATIONS

---

In the Vedic system there are general methods that always work (see Chapter 3 for general multiplication) and there are special methods which can be applied in special cases.

Here we look at a very powerful special method that works when the numbers are near a base number (like 10, 100, 1000 etc.) or a multiple of a base number.

In just a few minutes you will be able to easily tackle multiplications like:  
**789789 × 999997.**

**A** Numbers below a base number.

\* **88 × 97 = 8536**

We notice here that both numbers being multiplied are close to 100:  
88 is 12 below 100, and 97 is 3 below it.

$$\begin{array}{r} -12 \quad -3 \\ 88 \times 97 = \underline{85/36} \end{array}$$

The deficiencies (12 and 3) have been written above the numbers (*on the flag* – a sub-sutra), the minus signs indicating that the numbers are below 100.

The answer 8536 is in two parts: 85 and 36.  
The 85 is found by taking one of the deficiencies from the other number.

That is:  $88 - 3 = 85$  or  $97 - 12 = 85$  (whichever you like),

and the 36 is simply the product of the deficiencies:  $12 \times 3 = 36$ .  
It could hardly be easier.

\* Similarly, **93 × 96 = 8928**

$$\begin{array}{r} -7 \quad -4 \\ 93 \times 96 = \underline{89/28} \end{array}$$

The differences from 100 are 7 and 4,  
 $93 - 4 = 89$  or  $96 - 7 = 89$ ,  
and  $7 \times 4 = 28$ .




On the flag

\* Also  $98 \times 97 = \underline{9506}$

Note the zero inserted here: the numbers being multiplied are near to 100, so two digits are required on the right-hand part of the answer, as in the other examples.

\*  $89 \times 89 = 7 \overset{1}{8} 2 1 = 7921$

Here the numbers are each 11 below 100, and  $11 \times 11 = 121$ , a 3-figure number. The hundreds digit of this is therefore carried over to the left.

 Try these:

<b>a</b> 94 × 98	<b>b</b> 97 × 89	<b>c</b> 87 × 99	<b>d</b> 89 × 98
<b>e</b> 87 × 95	<b>f</b> 88 × 96	<b>g</b> 88 × 88	<b>h</b> 97 × 97

**B** Well, all these numbers are close to 100. But the numbers can be near any other base:

\*  $7 \times 8 = 56$

$$7 \times 8 = \underline{56}$$

Multiplication tables above  $5 \times 5$  are not essential in the Vedic system. Here 7 is 3 below the base of 10, and 8 is 2 below:  $7 - 2 = 5$  and  $3 \times 2 = 6$ , therefore  $7 \times 8 = 56$ .

*Like the crest of the peacock,  
like the gem on the head of a snake,  
so is mathematics at the head of all knowledge.”  
from the Jyotish Vedanga*

\*  $876 \times 998 = 874/248$

$$876 \times 998 = \overset{-124}{876} \times \overset{-2}{998} = \underline{874/248}$$

The base here is 1000.

So we need to know how much 876 is below 1000.

But this is given by *All from 9 and the Last from 10* described in the last chapter:  $1000 - 876 = 124$ .

Then,  $876 - 2 = 874$ ,  
and  $124 \times 2 = 248$ .

\* Similarly,  $\overset{-210211}{789789} \times \overset{-3}{999997} = \underline{789786/630633}$

where  $789789 - 3 = 789786$ , and  $210211 \times 3 = 630633$ .



Try these:

a  $888 \times 998$

b  $767 \times 998$

c  $989 \times 996$

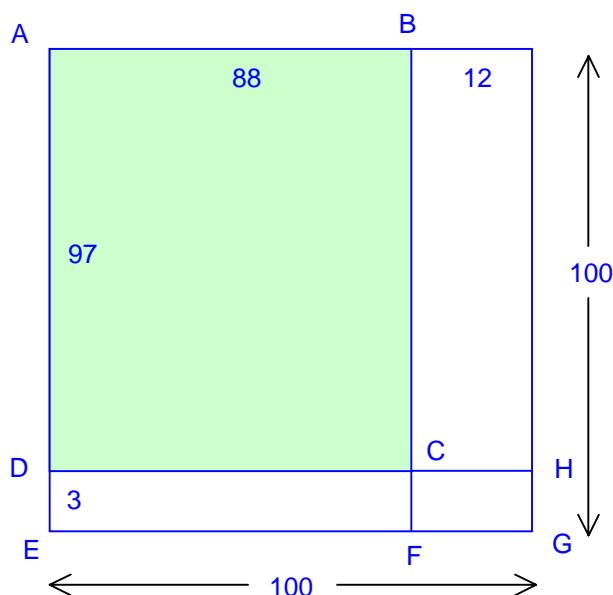
d  $8888 \times 9996$

e  $6999 \times 9997$

f  $66969 \times 99999$

### C A geometrical proof:

Here is a geometrical proof that  $88 \times 97 = 8536$ .



We start with a 100 by 100 square.

Then the product  $88 \times 97$  is the area of the rectangle ABCD (in blue).

But  $ABCD = ABFE - DHGE + CHGF$ .

$$\begin{aligned} \text{So } 88 \times 97 &= 88 \times 100 - 3 \times 100 + 3 \times 12 \\ &= 8800 - 300 + 36 \\ &= 8536 \end{aligned}$$

**D** But this is not all: if the numbers are above the base number the method still works and is even easier:

\*  $103 \times 104 = 10712$

$$103 \times 104 = \overset{+3}{103} \times \overset{+4}{104} = \underline{10712}$$

The numbers are near 100 and above it, so we use +3 and +4 (positive now because the numbers are above the base).

Then we **cross-add**:  $103 + 4 = 107$  or  $104 + 3 = 107$ , and  $3 \times 4 = 12$ .

\* Similarly:  $12 \times 13 = 156$  ( $12+3=15$ ,  $2 \times 3=6$ )

$$1234 \times 1003 = 1237702 \quad (1234+3=1237, 234 \times 3=702)$$

$$10021 \times 10002 = 100230042 \quad (10021+2=10023, 0021 \times 2=0042)$$



Try these:

**a**  $133 \times 103$

**b**  $107 \times 108$

**c**  $171 \times 101$

**d**  $102 \times 104$

**e**  $125 \times 105$

**f**  $14 \times 12$

**g**  $11112 \times 10003$

**h**  $1022 \times 1004$

*"That vast book which stands forever open before our eyes, the universe, cannot be read until we have learned the language and become familiar with the characters in which it is written. It is written in mathematical language, without which means it is humanly impossible to comprehend a single word"*  
Galileo Galilei

**E** But suppose one number is above the base and the other number is below it.

Can the method still be used?

The answer is yes, most certainly.

$$* \mathbf{124 \times 98 = 122\overline{48} = 12152}$$

Here the base is 100 and the differences from 100 are +24 and -2.

$$124 \times 98 = 122\overline{48} = \underline{12152}$$

Applying the usual procedure we find  $124 - 2 = 122$

$$\text{or } 98 + 24 = 122.$$

So 122 is the left-hand part of the answer.

Then multiplying +24 and -2 we get -48, written  $\overline{48}$  (since a plus times a minus gives a minus). This gives the answer as  $122\overline{48}$ .

To remove the negative portion of the answer we just take 48 from one of the hundreds in the hundreds column. This simply means reducing the hundreds column by 1 and applying *All From 9 and the Last From 10* to 48.

Thus 122 becomes 121 and  $\overline{48}$  becomes 52. (We did this in Chapter 1)

So  $122\overline{48} = 12152$ .

$$* \mathbf{1003 \times 987 = 990/\overline{039} = \underline{989/961}}$$

Similarly, we first get  $1003 - 13 = 990$  or  $987 + 3 = 990$ ,

and  $+3 \times -13 = \overline{039}$  (3 figures required here as the base is 1000).

Then 990 is reduced by 1 to 989, and applying the Sutra to 039 gives 961.



Try these:

**a**  $104 \times 91$

**b**  $94 \times 109$

**c**  $103 \times 98$

**d**  $92 \times 112$

**e**  $91 \times 111$

**f**  $991 \times 1005$

**g**  $987 \times 1006$

**h**  $992 \times 1111$

**F** Let's look at one more extension of this method: where the numbers are not near a base number, but are near a multiple of a base number, like 30, 200, 6000.

In fact we use a sub-sutra called *Proportionately*.

$$\ast \overset{+13}{213} \times \overset{+3}{203} = 2 \times 216/39 = \mathbf{43239}$$

We observe here that the numbers are not near any of bases used before: 10, 100, 1000 etc. But they are close to 200, and are 13 and 3 above 200.

The usual procedure gives us 216/39 (213+3=216, 13×3=39).  
Now since our base is 200 which is 100×2 we multiply only the left-hand part of the answer by 2 to get 43239.

$$\ast \overset{-1}{29} \times \overset{-2}{28} = 3 \times 27/2 = \mathbf{812}$$

Similarly here the numbers are just below 30.  
So the deficiencies are -1 and -2.  
29-2 = 27, 1×2 = 2 and 3×27 = 81.



Try these:

**a** 204 × 207

**b** 41 × 42

**c** 321 × 303

**d** 2003 × 2008

**e** 48 × 47

**f** 188 × 196

There are many other ways of using this far-ranging and useful method (see References ).

### 3

## VERTICAL AND CROSSWISE PATTERNS

---

The Vedic Sutra *Vertically and Crosswise* enables us to multiply any two numbers together in one line (from left to right or from right to left) no matter how big the numbers are.

**A** Let us start with multiplying 2-figure numbers.

\*  $21 \times 23 = 483$

we put the numbers one below the other and multiply:

- A. vertically on the right,
- B. then crosswise and add,
- C. and then vertically on the left:

$$\begin{array}{r} 2 \quad 1 \\ | \times | \\ \hline 2 \quad 3 \times \\ \hline 4 \quad 8 \quad 3 \end{array}$$

- A. multiply vertically in the right-hand column:  $1 \times 3 = 3$ ,  
so 3 is the last figure of the answer.

$$\begin{array}{r} 2 \quad 1 \\ \quad | \\ \hline 2 \quad 3 \times \\ \hline 4 \quad 8 \quad 3 \end{array}$$

- B. multiply crosswise and add:  
 $2 \times 3 = 6$ ,  
 $1 \times 2 = 2$ ,  $6 + 2 = 8$ ,  
so 8 is the middle figure of the answer.

$$\begin{array}{r} 2 \quad 1 \\ \quad \times \\ \hline 2 \quad 3 \times \\ \hline 4 \quad 8 \quad 3 \end{array}$$

- C. multiply vertically in the left-hand column:  $2 \times 2 = 4$ ,  
4 is the first figure of the answer.

$$\begin{array}{r} 2 \quad 1 \\ | \\ \hline 2 \quad 3 \times \\ \hline 4 \quad 8 \quad 3 \end{array}$$

That's all there is to it!



Try these:

<b>a</b> $\begin{array}{r} 22 \\ 31 \times \\ \hline \end{array}$	<b>b</b> $\begin{array}{r} 21 \\ 31 \times \\ \hline \end{array}$	<b>c</b> $\begin{array}{r} 21 \\ 22 \times \\ \hline \end{array}$	<b>d</b> $\begin{array}{r} 61 \\ 31 \times \\ \hline \end{array}$	<b>e</b> $\begin{array}{r} 32 \\ 21 \times \\ \hline \end{array}$	<b>f</b> $\begin{array}{r} 31 \\ 31 \times \\ \hline \end{array}$	<b>g</b> $\begin{array}{r} 13 \\ 13 \\ \hline \end{array}$
—	—	—	—	—	—	—



The previous examples involved no carry figures, so let us consider this next.

$$\begin{array}{r} * \quad 2 \ 3 \\ \quad 4 \ 1 \times \\ \hline 9 \ 4 \ 3 \\ \hline \end{array}$$


The 3 steps give us:  $3 \times 1 = 3$ ,  
 $2 \times 1 + 3 \times 4 = 14$ , so we carry 1 to the left as shown.  
 $2 \times 4 = 8$ , and  $8 + \text{the carried } 1 = 9$ .

So  $23 \times 41 = 943$ .

$$\begin{array}{r} * \quad 2 \ 3 \\ \quad 3 \ 4 \times \\ \hline 7 \ 8 \ 2 \\ \hline \end{array}$$

We have  $3 \times 4 = 12$ , so carry 1.  
 Crosswise gives 17, add carried 1 to get 18, so carry 1 again.  
 Then vertically gives  $2 \times 3 = 6$  and  $6 + 1 = 7$ .

$$\begin{array}{r} * \quad 3 \ 3 \\ \quad 4 \ 4 \\ \hline 1 \ 4 \ 5 \ 2 \\ \hline \end{array}$$

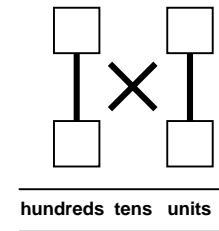
 Multiply the following:

<b>a</b> $\begin{array}{r} 2 \ 1 \\ \hline 4 \ 7 \\ \hline \end{array}$	<b>b</b> $\begin{array}{r} 2 \ 3 \\ \hline 4 \ 3 \\ \hline \end{array}$	<b>c</b> $\begin{array}{r} 2 \ 4 \\ \hline 2 \ 9 \\ \hline \end{array}$	<b>d</b> $\begin{array}{r} 2 \ 2 \\ \hline 2 \ 8 \\ \hline \end{array}$	<b>e</b> $\begin{array}{r} 2 \ 2 \\ \hline 5 \ 3 \\ \hline \end{array}$	<b>f</b> $\begin{array}{r} 3 \ 1 \\ \hline 3 \ 6 \\ \hline \end{array}$
<b>g</b> $\begin{array}{r} 2 \ 2 \\ \hline 5 \ 6 \\ \hline \end{array}$	<b>h</b> $\begin{array}{r} 3 \ 1 \\ \hline 7 \ 2 \\ \hline \end{array}$	<b>i</b> $\begin{array}{r} 4 \ 4 \\ \hline 5 \ 3 \\ \hline \end{array}$	<b>j</b> $\begin{array}{r} 3 \ 3 \\ \hline 8 \ 4 \\ \hline \end{array}$	<b>k</b> $\begin{array}{r} 3 \ 3 \\ \hline 6 \ 9 \\ \hline \end{array}$	<b>l</b> $\begin{array}{r} 3 \ 4 \\ \hline 4 \ 2 \\ \hline \end{array}$

*"All the effects of nature are only the mathematical consequences of a small number of immutable laws."  
 Marquis de Pierre Simon Laplace*

**Explanation** It is easy to understand how this method works.

The vertical product on the right multiplies units by units and so gives the number of units in the answer. The crosswise operation multiplies tens by units and units by tens and so gives the number of tens in the answer. And the vertical product on the left multiplies tens by tens and gives the number of hundreds in the answer.



**B** Not only is the method versatile and easy to do and understand but the same pattern will multiply algebraic expressions. By contrast the current methods for multiplying numbers and algebraic expressions are quite different and do not have the simple Vedic pattern.

\* Multiply  $(2x + 5)(3x + 2)$ .

$$\begin{array}{r} 2x \quad + 5 \\ 3x \quad + 2 \\ \hline 6x^2 + 19x + 10 \end{array}$$

Vertically on the right:  $5 \times 2 = 10$ .  
Crosswise:  $4x + 15x = 19x$ .  
Vertically on the left:  $2x \times 3x = 6x^2$ .

The Vedic method neatly collects like terms for us:  $(4x + 15x)$ .

\* Multiply  $(2x - 3)(3x + 4)$ .

This is very similar:  
And  $-3 \times 4 = -12$ ,  
Crosswise:  $8x - 9x = -1x$  or  $-x$ .  
 $2x \times 3x = 6x^2$ .

$$\begin{array}{r} 2x \quad - 3 \\ 3x \quad + 4 \\ \hline 6x^2 - x - 12 \end{array}$$



Try these:

**a**  $(2x + 5)(x + 4)$

**b**  $(x + 8)(3x + 11)$

**c**  $(2x + 1)(2x + 20)$

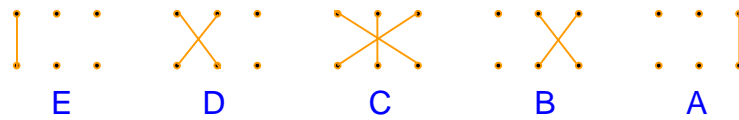
**d**  $(x - 3)(x - 3)$

**e**  $(2x - 3)(x + 4)$

**f**  $(2x - 3)(3x + 6)$

**C** Now let's look at multiplying longer numbers: three-figure numbers.

We may summarise the five steps (A, B, C, D, E) as follows:



The six dots shown at each step represent the two three-figure numbers.

\*  $504 \times 321 = 161784$ .

$$\begin{array}{r} 504 \\ 321 \\ \hline 161784 \\ \hline \end{array}$$

Following the pattern:

A Vertically on the right,  $4 \times 1 = 4$ .

$$\begin{array}{r} 504 \\ 321 \\ \hline 4 \\ \hline \end{array}$$

B Then crosswise on the right,  $0 \times 1 + 4 \times 2 = 8$ .

$$\begin{array}{r} 504 \\ \quad \times \\ \hline 321 \\ \hline 84 \\ \hline \end{array}$$

C Next we take 3 products and add them up,  $5 \times 1 + 0 \times 2 + 4 \times 3 = 17$ .

(actually we are gathering up the hundreds by multiplying hundreds by units, tens by tens and units by hundreds)

$$\begin{array}{r} 504 \\ \quad \times \\ \hline 321 \\ \hline 784 \\ \hline \end{array}$$

D Next we multiply crosswise on the left,  $5 \times 2 + 0 \times 3 = 10$ , and adding the carried 1 gives 11.

$$\begin{array}{r} 504 \\ \quad \times \\ \hline 321 \\ \hline 1784 \\ \hline \end{array}$$

E Finally, vertically on the left,  $5 \times 3 = 15$ , + carried 1 gives 16.

$$\begin{array}{r} 504 \\ \quad \times \\ \hline 321 \\ \hline 161784 \\ \hline \end{array}$$

Note the symmetry in the 5 steps:

first there is 1 product, then 2, then 3, then 2, then 1.

\*  $123 \times 45 = 5535$

To use our method for multiplying 3-figure numbers we can put 045 for 45.

$$\begin{array}{r} 123 \\ \underline{045} \\ 5535 \\ \underline{1230} \\ 5535 \end{array}$$

 Try these:

**a**  $\begin{array}{r} 212 \\ \underline{414} \\ \hline \end{array}$

**b**  $\begin{array}{r} 222 \\ \underline{333} \\ \hline \end{array}$

**c**  $\begin{array}{r} 105 \\ \underline{507} \\ \hline \end{array}$

**d**  $\begin{array}{r} 106 \\ \underline{222} \\ \hline \end{array}$

**e**  $\begin{array}{r} 515 \\ \underline{555} \\ \hline \end{array}$


# 4 SQUARING

---

The *Vertically and Crosswise* formula simplifies nicely when the numbers being multiplied are the same, and so gives us a very easy method for squaring numbers.

**A** We will use the term **Duplex**, D, to denote:

for 1 figure D is its square, e.g.  $D(4) = 4^2 = 16$ ;  
 for 2 figures D is twice their product, e.g.  $D(43) = 2 \times 4 \times 3 = 24$ ;  
 for 3 figures D is twice the product of the outer pair + the square of the middle digit, e.g.  $D(137) = 2 \times 1 \times 7 + 3^2 = 23$ ;  
 for 4 figures D is twice the product of the outer pair + twice the product of the inner pair, e.g.  $D(1034) = 2 \times 1 \times 4 + 2 \times 0 \times 3 = 8$ ;  
 $D(10345) = 2 \times 1 \times 5 + 2 \times 0 \times 4 + 3^2 = 19$ ;  
 and so on, but in fact we will use only the first three above in this book.

 Find the duplexes of these numbers:

<b>a</b> 5	<b>b</b> 2	<b>c</b> 63	<b>d</b> 16
<b>e</b> 616	<b>f</b> 801	<b>g</b> 256	

**B** The square of any number is just the total of its Duplexes.

\*  $43^2 = 1849$

So to square 43: we find the duplex of the 3 on the right,  
 then the duplex 43,  
 then the duplex of the 4 on the left.

$$43^2 = 1\ 8\ 24\ 9$$

D(3) = 9, that's the last figure:                   9  
 D(43) = 24, put down 4 and carry 2:        24 9  
 D(4) = 16, 16 + carried 2 = 18:           18 24 9

\*  $64^2 = 4096$

$$\begin{array}{r}
 D(4) = 16: \qquad \qquad \qquad 16 \\
 D(64) = 48: \qquad \qquad \qquad 49 \ 16 \\
 D(6) = 36: \qquad \qquad 40 \ 49 \ 16
 \end{array}$$



Try these:

a 31      b 14      c 41      d 26      e 66      f 81      g 56

### C And squaring algebraic expressions is just as easy:

\* Find  $(2x + 3)^2$ .

There are three Duplexes:     $D(3) = 9$ ,  
      $D(2x+3) = 2 \times 2x \times 3 = 12x$ ,  
      $D(2x) = 4x^2$ .

So  $(2x + 3)^2 = 4x^2 + 12x + 9$



Square the following:

h  $(3x + 4)$       i  $(5y + 2)$       j  $(2x - 1)$       k  $(x + 7)$

### D For squaring 3-figure numbers we need to include all the five duplexes:

\*  $341^2 = 116281$

$$\begin{array}{r}
 D(1) = 1: \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad 1 \\
 D(41) = 8: \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad 8 \ 1 \\
 D(341) = 22: \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad 2 \ 2 \ 8 \ 1 \\
 D(34) = 24, \text{ add carried } 2: \qquad \qquad \qquad \qquad \qquad \qquad 2 \ 6 \ 2 \ 8 \ 1 \\
 D(3) = 9, \text{ add carried } 2: \qquad \qquad \qquad 1 \ 1 \ 6 \ 2 \ 8 \ 1
 \end{array}$$

In just the same way we can square expressions like:  
( $x^2 + 3x + 2$ ) or ( $2x + 3y + 5$ ).



Square the following:

a 432

b 313

c 723

d ( $2x + 3y + 5$ )

**D** We could look here also at a special way of squaring numbers that are close to a base number (base numbers are 10, 100, etc.). This come under the Sutra *By the Deficiency*.

\*  $96^2 = 92/16$

96 is obviously close to 100 and is 4 below it (4 deficient).  
We therefore reduce the 92 by this number to get 92 as the first part of the answer.

To get the last part we just square the deficiency:  $4^2 = 16$ .

\*  $107^2 = 114/49$

Similarly here 107 is 7 over 100 so we increase 107 by another 7 to get 114.

Then  $7^2 = 49$ .

*“The construction of the  
physical and moral world alike  
is based on eternal numbers”  
St. Augustine*

\*  $993^2 = 986/049$

Here the base is 1000 so we expect 3 figures in each part of the answer. The deficiency is 7, reduce 993 by 7 to get the 986, and square 7 and put down 049 (the base is 1000, so 3 figures are needed).



Square the following:

**a** 94

**b** 98

**c** 88

**d** 106

**e** 103

**f** 996

**g** 988

**h** 1012

**i** 1001

**j** 10098



## 5 LEFT TO RIGHT

---

**A** Now here you will see some more of the flexibility and versatility of the Vedic system.

The current 'system' always calculates in one direction: for addition, subtraction and multiplication it goes right to left and for division and square roots it goes left to right.

But in the Vedic system we can work either way, and this has a number of important implications.

First it means our mental calculations are easier because we normally read from left to right, pronounce numbers from left to right and think of numbers in this order too.

Secondly, we can get at the most significant figures (the first figures) in a calculation first, not last.

And thirdly, it means that since all the basic operations, addition, subtraction, multiplication, division, square roots etc. can be done from left to right, we can combine the operations, and, for example, square two numbers, add them and take the square root, all in one line. We can also calculate trigonometric functions and their inverses, solve polynomial and transcendental equations and so on (see References 2, 3).

✱ **36 + 98 = 134**

$$\begin{array}{r} 36 \\ 98 \\ \hline 1_2 \end{array} +$$

In this addition sum, if we add the left-hand column we get  $3 + 9 = 12$ . Since there will be a carried figure from the next column that will affect this total, we put down only the 1 and carry the 2 forwards as shown.

$$\begin{array}{r} 36 \\ 98 \\ \hline 1_2 34 \end{array} +$$

The next column adds up to 14, We add the carried 2, **as 20**, to this to get 34 which we put down.

You can see that the carried 2 is really 20 because in the left column we are adding 30 and 90, which is 120.

In working from left to right any carry figure takes ten times its value when used in the next column.

\*  $765 + 618 = 1383$


$$\begin{array}{r} 765 \\ \underline{618} + \\ 1_3 \end{array}$$

Here we get  $7+6 = 13$  in the left-hand column.

The middle column adds up to 7, We add the carried 3, **as 30**, to this to get 37 and put down 3<sub>7</sub> as shown.

$$\begin{array}{r} 765 \\ \underline{618} + \\ \underline{1_3 3_7 8} 3 \end{array}$$

Then the right-hand column adds up to 13, to which we add the carried 7, **as 70**, to get 83: and this goes down to complete the answer.

 Add these:

<b>a</b> $\begin{array}{r} 86 \\ \underline{77} \\ \hline \end{array}$	<b>b</b> $\begin{array}{r} 47 \\ \underline{88} \\ \hline \end{array}$	<b>c</b> $\begin{array}{r} 73 \\ \underline{64} \\ \hline \end{array}$	<b>d</b> $\begin{array}{r} 678 \\ \underline{787} \\ \hline \end{array}$	<b>e</b> $\begin{array}{r} 836 \\ \underline{627} \\ \hline \end{array}$
--	--	--	--	--

**B** Multiplication is very similar.

\*  $3457 \times 8 = 27656$

$$\begin{array}{r} 3457 \\ \underline{\phantom{0}8 \times} \\ \underline{2_4 7_2 6_0 5} 6 \end{array}$$

We begin on the left:  $3 \times 8 = 24$ , put as shown;  
 $4 \times 8 = 32$ , add the carried 4, **as 40**,  $32 + 40 = 72$ ;  
 $5 \times 8 = 40$ , add the carried 2, **as 20**,  $40 + 20 = 60$ ;  
 $7 \times 8 = 56$ ,  $56 + 0 = 56$ , put as shown.

\*  $138 \times 4 = 0_4 5_2 5_2 = 552$ .

In the previous example we started with a 2-figure product (24) on the left. Here we initially get  $1 \times 4 = 4$ , a single figure number. So we put down 0<sub>4</sub> as shown.

*"Number rules the universe"*  
Pythagoras

 Try these:

$$\begin{array}{r} \text{a } 27 \\ \underline{3} \times \\ \hline \end{array}$$

$$\begin{array}{r} \text{b } 92 \\ \underline{7} \times \\ \hline \end{array}$$

$$\begin{array}{r} \text{c } 26 \\ \underline{6} \times \\ \hline \end{array}$$

$$\begin{array}{r} \text{d } 642 \\ \underline{4} \times \\ \hline \end{array}$$

$$\begin{array}{r} \text{e } 256 \\ \underline{3} \times \\ \hline \end{array}$$

You will recall the *Vertically and Crosswise* method from Chapter 3 for multiplying 2-figure numbers.

If you are not sure just go back and remind yourself now. Here is how it works from left to right.

\*  $63 \times 74 = 4662$

$$\begin{array}{r} 63 \\ \underline{74} \\ 4_2 6_5 62 \end{array}$$

- A)  $6 \times 7 = 42$ , put down 4 and carry 2.
- B)  $(6 \times 4) + (3 \times 7) = 45$ , add carried 2, as 20,  $45 + 20 = 65$ , put down 6, carry 5.
- C)  $3 \times 4 = 12$ , add carried 5, as 50,  $12 + 50 = 62$ , put down 62.

Sometimes however a figure needs to be carried leftwards as shown next.

\*  $71 \times 74 = 5254$

$$\begin{array}{r} 71 \\ \underline{74} \\ 4_9^1 2_5 54 \\ = \underline{5254} \end{array}$$

- A)  $7 \times 7 = 49$ , put down 4 and carry 9.
- B)  $(7 \times 4) + (1 \times 7) = 35$ , and adding the carried 9, as 90 gives  $35 + 90 = 125$ .  
The 1 here is carried leftwards to join the 4 and the 2 and 5 are placed as usual.
- C) then  $1 \times 4 = 4$ , add carried 5, as 50,  $4 + 50 = 54$ : put down 54.

This comes up in **k** below.

 Multiply these:

$$\begin{array}{r} \text{f } 32 \\ \underline{46} \\ \hline \end{array}$$

$$\begin{array}{r} \text{g } 31 \\ \underline{72} \\ \hline \end{array}$$

$$\begin{array}{r} \text{h } 44 \\ \underline{53} \\ \hline \end{array}$$

$$\begin{array}{r} \text{i } 33 \\ \underline{84} \\ \hline \end{array}$$

$$\begin{array}{r} \text{j } 34 \\ \underline{22} \\ \hline \end{array}$$

$$\begin{array}{r} \text{k } 34 \\ \underline{64} \\ \hline \end{array}$$

**C** Squaring too we can do from the left. You may like to remind yourself of the squaring method (using duplexes) from the last chapter before proceeding.

\*  $43^2 = 1849$

Working from left to right there are three duplexes in 43:  
D(4), D(43) and D(3).

$$\begin{array}{l} D(4) = 4^2 = \mathbf{16}: \\ D(43) = 2 \times (4 \times 3) = \mathbf{24}, 24 + 60 = 84: \\ D(3) = 3^2 = \mathbf{9}, 9 + 40 = 49: \end{array} \quad \begin{array}{r} 1_6 \\ 1_6 8_4 \\ 43^2 = 1_6 8_4 49 \end{array}$$

\*  $64^2 = 4096$

$$\begin{array}{l} D(6) = \mathbf{36}: \\ D(64) = \mathbf{48}, 48 + 60 = 108: \\ D(4) = \mathbf{16}, 16 + 80 = 96: \end{array} \quad \begin{array}{r} 3_6 \\ 3_6^1 0_8 = 4_6 0_8 \text{ (note we must carry the 1} \\ \text{back to the 3 here)} \\ 64^2 = 4_6 0_8 96. \end{array}$$

\*  $441^2 = 194481$

Here we have a 3-figure number:

$$D(4) = 16, D(44) = 32, D(441) = 24, D(41) = 8, D(1) = 1.$$

$$\text{So } 441^2 = 1_6 9_2 4_4 4_8 81.$$



Square these numbers:

**a** 81

**b** 91

**c** 63

**d** 56

**e** 623

**f** 234

## 6 SQUARE ROOTS

---

**A** The squaring method from Chapter 4 can also be reversed to give a one-line method of finding the square root of any number.

\*  $\sqrt{2704} = 52$

This means we are looking for a number whose square is 2704.

We can see the first figure must be **5** because  $50^2 = 2500$  and  $60^2 = 3600$ ,  
and 2704 comes between 2500 and 3600.

Since  $50^2 = 2500$  and 2704 starts with 27, there is a remainder of 2 which we place as shown below to give 20 in the tens place:

$$\sqrt{27_204} = 5?$$

To get the last figure (?) we just divide this 20 (<sub>2</sub>0) by **twice the first answer figure**. That is  $20 \div 10 = 2$ .

So we get  $\sqrt{2704} = 52$ .

This is quite straight forward: we decide the first figure of the answer (5 in the above example) and put it down and also the remainder (2). Then we divide the next 2-figure number (20) by twice the answer figure (twice 5).

\*  $\sqrt{2116} = 46$

Seeing 2116 starts with 21, the first figure must be **4** as  $4^2 = 16$ .  
And there will therefore be a remainder of 5 as  $21 - 16 = 5$ .

So:  $\sqrt{21_516} = 4?$

Now divide 51 (<sub>5</sub>1) by 8 (twice 4).  
This gives **6**, so  $\sqrt{2116} = 46$ .



square roots

☞ Find the square root:

a 1936

b 5184

c 8836

d 4489

\* Look again at  $\sqrt{2116}$ .

When we had  $\sqrt{21_5 16} = 4?$  we divided 51 by 8 and got 6.

In fact in dividing 51 by 8 there is a remainder of 3 which we could place as shown:

$$\sqrt{21_5 1_3 6} = 46.$$

Now the fact that the square of the last figure,  $6^2 = 36$  is seen at the end of  $\sqrt{21_5 1_3 6}$  confirms that the answer is exactly 46.

☞ Find the square root of these numbers and check they are exact:

e 3969

f 3136

g 5041

h 4356

*“Even the highest and farthest reaches of modern Western mathematics have not yet brought the Western world even to the threshold of Ancient Indian Vedic Mathematics.”*  
Professor Augustus De Morgan

\* Find  $\sqrt{1444}$ .

Here we see the first figure must be 3, so:

$$\sqrt{14_544} = 3?$$

If we now divide 54 by 6 we get 9.

But this is not right as  $\sqrt{14_54_04}$  is not 39 because we expect to see 81 (9 squared) left on the right and we have 04.

Another reason for knowing 39 is wrong is that  $39^2$  must end with a 1 because  $9^2 = 81$ .

This tells us that the 9 is too big and must be reduced to 8.

$$\text{So } \sqrt{14_54_64} = 38.$$

By saying that  $54 \div 6 = 8$  remainder 6 we get 64 at the end of 1444 and the square of 8 is indeed 64.



Find the square root of these numbers and check they are exact:

a 784

b 2401

c 841

**B** Although all the above were perfect squares so that the answer could be found exactly, the same method will give the first two figures of the answer if the number given is not a perfect square.

For example:

$$* \sqrt{3000} \approx 55$$



a perfect square

The symbol  $\approx$  means 'approximately equal to'

We get the first figure, 5, as before and there is a remainder of 5, so:

$$\sqrt{30_500} \approx 5?$$

Then divide 50 by twice the first figure, as before:  $50 \div 10 = 5$ .

The method is just the same as before and usually gives the first two figures of the answer.

Of course this Vedic method can be developed to give as many figures as may be required, but we will not go into that here (see References 1, 2, 4).

$$* \sqrt{6789} \approx 82$$

8 is clearly the first figure with 3 remainder, and  $38 \div 16$  gives the 2.



Find the approximate square root of these numbers:

**a** 4000

**b** 2828

**c** 42.42

**d** 32



# 7

## THE CROWNING GEM

---

**A** The man who rediscovered the Vedic system, Sri Bharati Krsna Tirthaji, from the ancient Vedic texts gives this beautiful method for dividing any number by any other number.

And it is all done in one line!

\* Suppose we want to **divide 219 by 52**.

We can set the sum out like this:

$$\begin{array}{r|l} 2 & 219 \\ 5 & 4 \quad 11 \\ \hline & \end{array}$$

The **divisor**, **52**, is written with the 2 raised up, *On the Flag*, and a vertical line is drawn one figure from the right-hand end to separate the answer, 4, from the remainder, 11.

The steps are:

- A)** 5 into 21 goes 4 remainder 1, as shown.
- B)** Answer digit 4 multiplied by the flagged 2 gives 8, and this 8 taken from 19 (19) leaves the remainder of 11, as shown.

### Explanation

We need to know how many 52's there are in 219. Looking at the first figures we see that since 5 goes into 21 four times we can expect four 52's in 219.

We now take four 52's from 219 to see what is left. Taking four 50's from 219 leaves 19 and we need to take four 2's away as well. This leaves a remainder of 11.

\* **Divide 321 by 63.**

We set the sum out:

$$\begin{array}{r|rr|l} 63 & 32 & 1 & \\ \hline & 156 & & \\ \hline & & 6 & \end{array} = 5 \text{ remainder } 6$$

6 into 32 goes 5 remainder 2, as shown, and answer, 5, multiplied by the flagged 3 gives 15, which we take from the 21 to leave the remainder of 6.



Try these:

a  $103 \div 43$

b  $234 \div 54$

c  $74 \div 23$

d  $504 \div 72$

**B** If we are dividing into a longer number this method is simply extended.

\*  **$17496 \div 72 = 243$  remainder 0.**

$$\begin{array}{r|rrrr|l} 72 & 1749 & 6 & \\ \hline & & & & \\ \hline & & & & \end{array}$$

We set the sum out similarly, with the 2 of 72 raised on the flag. And we mark off the last digit by a vertical line to separate the remainder. In fact the number of figures we mark off at the right is always the number of figures we put on the flag (in our case, just one).

Then we divide 7 into 17 and put down 2 remainder 3 as shown below.

$$\begin{array}{r|rr|rl|l} 72 & 17 & 49 & 6 & \\ \hline & 14 & & & \\ \hline & & 3 & & \\ & & & & \end{array}$$

Note the diagonal: 2, 3, 4 which shows the 34 which is used in the next step.

We multiply the answer digit, 2, by the flag 2, to get 4, take this 4 from the 34 to get 30, which we then divide by 7: we put down 4 remainder 2:

$$\begin{array}{r|rr|rl|l} 72 & 17 & 49 & 6 & \\ \hline & 14 & 28 & & \\ \hline & & 3 & 2 & \\ & & & & \end{array}$$

Note the diagonal: 4, 2, 9.

Then we repeat the procedure: multiply last answer digit 4, by the flag, 2 to get 8. Take this 8 from 29 to get 21. And divide this 21 by 7 to get 3 remainder 0:

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 4 & 9 & 6 \\ 7 & & 3 & 2 & 0 & \\ \hline & 2 & 4 & 3 & & \end{array}$$

Finally we multiply the last answer digit, 3, by the flag, 2, to get 6.

We take this 6 from the 06 to get the remainder of 0, which we put down:

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 4 & 9 & 6 \\ 7 & & 3 & 2 & 0 & \\ \hline & 2 & 4 & 3 & 0 & \end{array}$$

We have been through this in some detail. In actual practice it is straightforward and quick.

Note how the calculation goes in **cycles**:

divide  
multiply  
subtract  
divide } repeated



Try these:

**a**  $19902 \div 62$

**b**  $23824 \div 51$

**c**  $92054 \div 63$

**d**  $2702 \div 54$

*“Number is an immediate emanation from the  
pure laws of thought”  
Dedekind*

**C** Now, it can happen that when we want to do the subtraction part we need to take away more than we have got. This is easily dealt with as shown in the next example.

\*  $33433 \div 54 = 619$  remainder 7.

If we proceed as before:

$$\begin{array}{r|rrrr|} 5 & 4 & 3 & 3 & 4 & 3 & 3 \\ & & & & 3 & 0 & \\ \hline & & & 6 & 2 & & \end{array}$$

We find we have to take 14 from 13, which means the answer is 7 rem  $\bar{1}$ .

After the first diagonal we take 24 from 34, which gives 10.  $10 \div 5$  gives 2 remainder 0, as shown.

But now we have to take 8 from 3, which will give a negative result.

Using negative numbers is an option, but an alternative is: instead of saying  $10 \div 5$  gives 2 remainder 0, say  $10 \div 5$  gives 1 remainder 5 (there is one five in ten and five left over).

$$\begin{array}{r|rrrr|} 5 & 4 & 3 & 3 & 4 & 3 & 3 \\ & & & & 3 & 5 & 4 \\ \hline & & & 6 & 1 & 9 & 7 \end{array}$$

This gives  $53 - 4$  at the next step and we then proceed as normal.



Try these:

**a**  $14018 \div 64$

**b**  $4712 \div 45$

**c**  $2602 \div 54$

**d**  $97 \div 28$

**D** Instead of giving a remainder we may wish to keep dividing. For that we just continue with the division process.

\* Find  $40342 \div 73$  to 5 decimal places.

Where before we put the vertical line to separate the remainder (between the 4 and the 2) we now put a decimal point in the answer line:

$$\begin{array}{r|l}
 3 & 40342.00000 \\
 7 & \phantom{4}535411362 \\
 \hline
 & 552.630137
 \end{array}$$

So  $40342 \div 73 = 552.63014$  to 5 decimal places.

We will not go through every step except to point out that when we come to the third diagonal, 2, 5, 2, we multiply answer 2 by flag, 3, to get 6, take this from 52 to get 46 and divide 46 by 7 to get 6 remainder 4, as shown.

So we simply continue in the same way for as long as we like.



Find to three decimal places:

**a**  $40342 \div 73$

**b**  $371426 \div 81$

**c**  $888 \div 61$

**d**  $17 \div 72$

*"God created the integers,  
 the rest is the work of man."  
 Kronecker*

**E** Finally, it is sometimes best to have a negative flag figure.

\* Find  $2345 \div 48$  to 2 decimal places.

Having the 8 on the flag here means the subtractions at each step would be large and we would consequently need to make the reductions described in section C.

To avoid this we use  $5\bar{2}$  ( $5\bar{2}$  means  $50 - 2$ ) rather than 48.

This will mean that we are always subtracting a negative number, which means **we add rather than subtract**:

$$\begin{array}{r|l} \bar{2} & 2\ 3\ 4\ 5.\ 0\ 0 \\ 5 & \quad \quad \quad 3\ 2\ 1\ 1 \\ \hline & 4\ 8.\ 8\ 5 \end{array}$$

$23 \div 5 = 4$  rem 3.

$4 \times \bar{2} = \bar{8}$ , so we add 8 to 34 to get 42:  $42 \div 5 = 8$  rem 2.

$8 \times \bar{2} = \bar{16}$ , so add 16 to 25 to get 41:  $41 \div 5 = 8$  rem 1.

Etc.

So  $2345 \div 48 = 48.85$  to 2 decimal places.



Find the 2 decimal places:

**a**  $432 \div 68$

**b**  $777 \div 47$

**c**  $300 \div 59$

**d**  $13 \div 79$

There are other useful variations that simplify the work further, and of course we have only covered divisions by 2-figure divisors whereas this method can divide numbers of any size, in one line every time (see References 1, 2, 4).

## 8 MATH OR MAGIC?

---

### A

- \* Suppose you want to convert the fraction  $\frac{1}{19}$  into its decimal form. The usual method would involve dividing by 19 repeatedly until the answer figures start to recur.

The answer can however be put down in one line by using the sutra

*By One More than the One Before.*

We divide by 2 (as 2 is the number one more than the 1 before the 9):

1 9  
↓  
one more than 1 is 2

In fact this 2 is called the “Ekadhika”.

So we begin by dividing the numerator (top number) of the fraction by the Ekadhika (2 here) and we just keep dividing by it.

So we start with 0 and a decimal point, then 1 (the numerator) divided by 2 (the Ekadhika) goes 0 remainder 1:

$$\frac{1}{19} = 0,10$$

Note carefully that we put the remainder 1 **before** the answer, 0.

We now have 10 (10) in front of us and we divide this by 2:

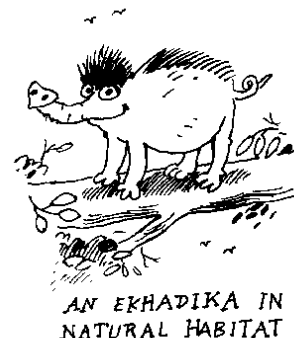
$$\frac{1}{19} = 0,105$$

We then divide this 5 by 2.  
This gives 2 remainder 1 so we now have:

$$\frac{1}{19} = 0,1052$$

Again we put the remainder, 1, before the 2.

Then we divide 12 by 2 and put down 6.



$$\frac{1}{19} = 0, \dot{0}5_1 263_1 15_1 7_1 89_1 47_1 3_1 6842\dot{1}$$

And so we continue, dividing the last answer by 2, putting down the result and prefixing any remainder.

Much better than dividing by 19!

When we reach 1 the decimal starts to repeat itself because if we continue we would be dividing 2 into 1 which is how we started. We therefore put a dot over the first and last figures of the 18-digit block to show that this repeats itself indefinitely.

Note that this method, as given so far, is for fractions where the denominator (bottom number) ends in a nine.

\* Convert  $\frac{11}{19}$  to a recurring decimal.

The Ekadhika is still **2** because we still have 19 in the denominator.

But we begin by dividing **2** into 11:  $\frac{11}{19} = 0,5$

**2** into 11 goes 5 remainder 1.

Next we divide **2** into 15:  $\frac{11}{19} = 0,5_1 7$



Continue the division until the decimal starts to repeat itself:

**a**  $\frac{11}{19} = 0,5_1 7$

\* Now let us take  $\frac{17}{29}$ .

Since we now have 29 our Ekadhika becomes 3 (one more than 2 is 3).

This means we start by dividing 17 by **3**, and keep on dividing by **3**: **3** into 17 goes 5 remainder 2.

$$\frac{17}{29} = 0,2_5$$

Then **3** into 25 goes 8 remainder 1; and **3** into 18 goes 6 and so on.



☞ Continue this division until the decimal starts to repeat itself:

a  $\frac{17}{29} = 0.25,86$

### C A SHORT CUT:

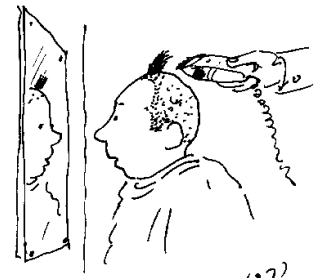
Incredibly enough even this small amount of work can be shortened!

Look again at the answer for  $\frac{1}{19}$ .

$$\frac{1}{19} = 0.052631578947368421$$

Here the 18 figures are in two rows of nine.  
You may spot a connection between the rows:

**each column adds up to nine.**



a short cut

This means that when we are half way through the decimal we can write down the second half from the first half, by taking all the numbers in the first half from 9!

But, you might ask, how do we know when we are half way through? The answer is to just take the numerator of the given fraction,  $\frac{1}{19}$ , from the denominator ( $19-1=18$ ) and when 18 comes up (18 appears at the end of the top row) you are half way through!

☞ b Confirm that this also works for  $\frac{11}{19}$ .

You should find that 8 (which is  $19-11$ ) appears half way through.

c Similarly check  $\frac{17}{29}$  which has 28 recurring figures.

d Find the recurring decimal for  $\frac{27}{39}$ . Your ekadhika will be 4 and look out for the half-way number (12).

e Find  $\frac{3}{49}$  (it starts  $\frac{3}{49} = 0.030$ )

In fact the half-way number does not always come up, but if it does then you can take all the numbers found so far from nine to get the rest.

 a Find the recurring decimal for  $\frac{29}{39}$ .

## D FROM RIGHT TO LEFT

The Vedic system is so versatile we can often work in either direction. Here we can also start from the right-hand end of a recurring decimal.

In fact the last figure in each of the examples you have done so far is equal to the last figure of the numerator of the given fraction. So  $\frac{1}{19}$  ended with a '1' and  $\frac{29}{39}$  ended with '29'. This must be so because, for example, we need 29 at the end of  $\frac{29}{39}$  in order for it to recur from there on.

\* So if we want to get  $\frac{1}{19}$  from right to left we start with the '1' at the right, and we **multiply** by the ekadhika, 2. And each figure we put down gets multiplied by 2 to get the next figure.

$$\frac{1}{19} = 0.\dot{1}05_1263_115_17_18_9_147_13_16842\dot{1}$$

We get  $1 \times 2 = 2$ ,  $2 \times 2 = 4$ ,  $4 \times 2 = 8$ ,  $8 \times 2 = 16$  written  $16$ .  
Then  $6 \times 2 +$  the carried 1 = 13,  
 $3 \times 2 +$  the carried 1 = 7.  
And so on.

So to find  $\frac{19}{39}$  you could start with 19 on the right and multiply by 4.

 b Find the recurring decimal for  $\frac{19}{39}$  working from right to left.

This method is not restricted to fractions with denominators ending in 9. It can be extended in various ways: a slight adjustment enables us to deal with denominators ending in 8, 7 or 6, and there is a complementary method for when the denominator ends in 1, which, with an adjustment allows for denominators ending in 2, 3 or 4.

## E PROPORTIONATELY AGAIN

Also, using the *Proportionately* formula we can change many fractions so that the denominator ends in 9.

\* Find the decimal for  $\frac{7}{13}$ .

The denominator here ends in a 3 (not a 9). But we can multiply top and bottom by 3 to get:  $\frac{7}{13} = \frac{21}{39}$  and 39 does end in 9. So we use an Ekadhika of 4 and divide 21 by 4 (looking out for the half-way number 18 also).

This gives us:  $\frac{7}{13} = \frac{21}{39} = 0.\dot{5}_3 3_1 8_4 6_1 \dot{1}$  (left to right).



Convert to recurring decimals:

**a**  $\frac{1}{7}$

**b**  $\frac{2}{13}$

**c**  $\frac{5}{23}$

**d**  $\frac{7}{33}$

**e**  $\frac{9}{11}$

**f**  $\frac{3}{17}$

Another application of *Proportionately* enables us to find answers two, three, four or more figures at a time. And there are other beautiful devices and variations that further simplify the work.

## 9 EQUATIONS

---

**A** The solution of equations comes under the Sutra *Transpose and Apply*. This is equivalent to the common method of “change the side, change the sign”. But the Vedic method is to find the answer in one go rather than write down every step of the process.

\* Solve  $5x - 4 = 36$ .

Using the Sutra we add 4 to 36 to get 40, then  $40 \div 5 = 8$ , so  $x = 8$ .

\* Solve  $\frac{x}{7} + 3 = 5$ .

Here we take 3 from 5 to get 2, then multiply 2 by 7, so  $x = 14$ .

\* Solve  $\frac{2x}{3} = 4$ .

Multiply 3 by 4 to get 12, then  $12 \div 2 = 6$ , so  $x = 6$ .

\* Solve  $\frac{3x+2}{4} = 8$ .

First  $8 \times 4 = 32$ , then  $32 - 2 = 30$ , then  $30 \div 3 = 10$  so  $x = 10$ .

 Try these:

**a**  $5x - 21 = 4$

**b**  $\frac{x}{3} + 4 = 6$

**c**  $\frac{x+4}{7} = 5$

**d**  $\frac{3x}{8} + 17 = 20$

**e**  $\frac{2x+1}{3} = 4$

*“An equation for me has no meaning  
unless it expresses a thought of God.”  
Srinivasa Ramanujan*

\* Solve  $5x + 3 = 3x + 17$

These equations can also be solved mentally. We can see how many x's there will be on the left and what the number on the right will be when we have transposed. Then we just divide the number on the right by the coefficient on the left.

So, in the example above we see there will be 2x on the left when the 5x is taken over, and that there will be 10 on the right when the 1 is taken over. Then we just divide 10 by 2 to get  $x = 5$ .

\* Solve  $7 - 2x = x - 5$

Here seeing the  $-2x$  it is best to collect the x terms on the right. Then the  $-2x$  will become  $+2x$  on the right, and the  $-5$  will be  $+5$  on the left. This gives  $12 = 3x$  (mentally). So  $x = 4$ .



Solve:

**a**  $7x - 5 = 4x + 10$

**b**  $5 + 4x = 13 + 2x$

**c**  $5x - 21 = x - 1$

**d**  $14 - 3x = x + 10$

**e**  $10x + 1 = 25 - 2x$

**f**  $2(3x + 1) = x + 27$

## B Quadratic Equations

Here we use the *Calculus Sutra*.

There is a simple relationship between the differential and the discriminant in a quadratic equation:

The differential is equal to the square root of the discriminant.

The **discriminant** of a quadratic expression is:

**the square of the coefficient of the middle term minus the product of twice the coefficient of the first ( $x^2$ ) term and twice the last term.**

That is, in the quadratic expression  $ax^2 + bx + c$ , the first differential is  $2ax + b$  and the discriminant is  $b^2 - 4ac$ .

So that  $2ax + b = \pm\sqrt{b^2 - 4ac}$ .

This is equivalent to the usual form of the formula for solving quadratic equations,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , but is much simpler.

\* Solve  $3x^2 + 8x - 3 = 0$ .

The differential is the square root of the discriminant gives:

$$6x + 8 = \pm\sqrt{100}.$$

So  $6x + 8 = \pm 10$ .

$6x + 8 = 10$  gives  $x = \frac{1}{3}$ .

And  $6x + 8 = -10$  gives  $x = -3$ .

 Try these:

**a**  $x^2 - 6x + 5 = 0$

**b**  $x^2 + 4x - 12 = 0$

**c**  $x^2 + 8x + 15 = 0$

## C Simultaneous Equations

We will not go into the general method for solving simultaneous equations (see Reference for that) here, but we can illustrate the Sutra *If One is in Ratio the Other One is Zero*, which easily solves equations of a special type.

\* Solve  $3x + 2y = 6,$   
 $9x + 5y = 18.$

We notice that the ratio of the x coefficients is the same as the ratio of the coefficients on the right-hand side:  $3:9 = 6:18$ .

This tells us that since x is in ratio the other one, y, is zero:  $y = 0$ .

If  $y = 0$  we can easily find x by putting  $y = 0$  in the first (or the second) equation:  $3x + 0 = 6$ .

Therefore  $x=2$ . So  $x = 2, y = 0$ .



Solve these:

a  $3x + 2y = 6$   
 $2x + 3y = 9$

b  $3x + 8y = 6$   
 $5x + 3y = 10$

c  $4x - y = 20$   
 $x + 5y = 5$

## D A special type of equation

\* Solve  $\frac{2x-3}{2x-5} = \frac{4x-9}{4x-7}$ .

This looks difficult and would normally be solved with a great deal of time and effort.

But it is a special type of equation (not recognized as such in the current system) that can be easily solved using

*If the Total is the Same that Total is Zero.*

It can be seen by mental cross-multiplication that the equation is linear ( $2x \times 4x = 4x \times 2x$ ), and according to the Sutra, since **the sum of the numerators = the sum of the denominators**, this total is zero.

That is  $(2x - 3) + (4x - 9) = 6x - 12$ , and also  $(2x - 5) + (4x - 7) = 6x - 12$ .

Therefore  $6x - 12 = 0$  and  **$x = 2$** .

\* Solve  $\frac{2x+5}{x+3} = \frac{6x+35}{3x+17}$ .

Here we have  $8x + 40$  as the sum of the numerators and  $4x + 20$  as the sum of the denominators. Dividing out the common factor we get  $x + 5$  for both (that is, we divide  $8x + 40$  by 8 and divide  $4x + 20$  by 4: both give  $x + 5$ ).

Therefore  $x + 5 = 0$  and  **$x = -5$** .

\* Solve  $\frac{2x-3}{x+4} = \frac{x-9}{2x-16}$ .

Here the total of the numerators and the denominators are both  $3x - 12$ . So  $3x - 12 = 0$  and  **$x = 4$** .

But mental cross-multiplication reveals that this equation is not linear, but quadratic.

However the samuccaya formula will give us the other solution also: the **difference between numerator and denominator** on either side is the same, and this we equate to zero.

$x - 7 = 0$ , therefore  $x = 7$  is the other solution.

 Try these:

**a**  $\frac{x+3}{x+4} = \frac{2x+5}{2x+4}$

**b**  $\frac{x+2}{2x+1} = \frac{x+1}{2x+5}$

**c**  $\frac{2x+1}{x+2} = \frac{2x-3}{3x-4}$

For a more thorough treatment of the Vedic approach to solving equations see References 1, 2.

*“Equations are more important to me,  
because politics is for the present, but an  
equation is something for eternity.”  
Albert Einstein*



# 10 SIXTEEN SUTRAS

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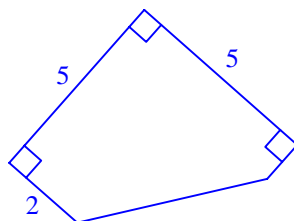
Here is a full list of the Sutras:

- 1 ***By One More than the One Before***
- 2 ***All from 9 and the Last from 10***
- 3 ***Vertically and Crosswise***
- 4 ***Transpose and Apply***
- 5 ***If the Total is the Same it is Zero***
- 6 ***If One is in Ratio the Other is Zero***
- 7 ***By Addition and by Subtraction***
- 8 ***By the Completion or Non-Completion***
- 9 ***Differential Calculus***
- 10 ***By the Deficiency***
- 11 ***Specific and General***
- 12 ***The Remainders by the Last Digit***
- 13 ***The Ultimate and Twice the Penultimate***
- 14 ***By One Less than the One Before***
- 15 ***The Product of the Sum***
- 16 ***All the Multipliers***

Not all of the sixteen Sutras on which the Vedic system rests have been shown in the previous chapters so for completeness let us end with a brief mention of those Sutras still not illustrated.

*"The Sutras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word 'mental!'"*  
*Sri Bharati Krsna Tirthaji*

Sutra 8, ***By the Completion or Non-Completion***. The technique called 'completing the square' for solving quadratic equations is an illustration of this Sutra. But the Vedic method extends to 'completing the cubic' and has many other applications (see Reference 1 or 3). For example to find the area of the figure below we could complete the square.



Sutra 11, ***Specific and General*** is useful for finding products and solving certain types of equation. It also applies when an average is used, as an average is a specific value representing a range of values. So to multiply  $57 \times 63$  we may note that the average is 60 so we find  $60^2 - 3^2 = 3600 - 9 = 3591$ . Here the 3 which we square and subtract is the difference of each number in the sum from the average (see Reference 3).

Sutra 12, ***The Remainders By the Last Digit*** is useful in converting fractions to decimals. For example if you convert  $\frac{1}{7}$  to a decimal by long division you will get the remainders 3, 2, 6, 4, 5, 1 . . . and if these are multiplied in turn by the last digit of the recurring decimal, which is 7, you get 21, 14, 42, 28, 35, 7 . . . The last digit of each of these gives the decimal for  $\frac{1}{7} = 0.142857 . . .$  The remainders and the last digit of the recurring decimal can be easily obtained (see Reference 1 or 3).

Sutra 13, ***The Ultimate and Twice the Penultimate*** can be used for testing if a number is divisible by 4. To find out if 9176 is divisible by 4 for example we add the ultimate (last figure) which is 6 and twice the penultimate, the 7. Since  $6 + \text{twice } 7 = 20$  and 20 is divisible by 4 we can say 9176 is divisible by 4.

Similarly, testing the number 27038 we find  $8 + \text{twice } 3$  which is 14. As 14 is not divisible by 4 neither is 27038 (see Reference 2 or 3).

Sutra 14, ***By One Less than the One Before*** is useful for multiplying by nines. For example to find  $777 \times 999$  we reduce the 777 by 1 to get 776 and then use *All from 9 and the last from 10* on 777. This gives  $777 \times 999 = 776/223$ .

Sutra 15, ***The Product of the Sum*** was illustrated in the Fun with Figures book for checking calculations using digit sums: we find the product of the digit sums in a multiplication sum to see if it is the same as the digit sum of the answer. Another example is Pythagoras' Theorem: the square on the hypotenuse is the sum of the squares on the other sides.

Sutra 16, ***All the Multipliers*** is useful for finding a number of arrangements of objects. For example the number of ways of arranging the three letters A, B, C is  $3 \times 2 \times 1 = 6$  ways.

## REFERENCES

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- 2 *Discover Vedic Mathematics* by Kenneth Williams, Inspiration Books, 1984. ISBN 1869932013
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Kenneth Williams and Mark Gaskell. ISBN 095317820X

For more information about books on Vedic Mathematics see:

[www.vedicmath.com](http://www.vedicmath.com)

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## ANSWERS

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**a** 83      **b** 71      **c** 83      **d** 61  
**e** 231    **f** 464    **g** 37      **h** 67

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**a** 25      **b** 37      **c** 239    **d** 127  
**e** 356    **f** 824    **g** 276    **h** 389

**i** 754    **j** 183    **k** 6511   **l** 72

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**a** 954      **b** 9273  
**c** 9916    **d** 9792

**e** 3754    **f** 662    **g** 42511   **h** 8198

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**a** 375      **b** 288      **c** 416      **d** 1758

**e** 2735    **f** 3716    **g** 3246

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**a** 9212    **b** 8633    **c** 8613    **d** 8722  
**e** 8265    **f** 8448    **g** 7744    **h** 9409

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**a** 886/224      **b** 765/466      **c** 985/044  
**d** 8884/4448    **e** 6996/9003    **f** 66968/33031

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**a** 136/99      **b** 115/56      **c** 172/71      **d** 106/08  
**e** 131/25      **f** 16/8      **g** 11115/3336    **h** 1026/088

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**e** 101/01    **f** 995/955    **g** 992/922    **h** 1102/112

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**a** 422/28      **b** 172/2      **c** 972/63  
**d** 4022/024    **e** 225/6      **f** 368/48

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**a** 682    **b** 651    **c** 462    **d** 1891    **e** 672    **f** 961    **g** 169

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**a** 987      **b** 989      **c** 696      **d** 616      **e** 1166      **f** 1116  
**g** 1232    **h** 2232    **i** 2332    **j** 2772    **k** 2277    **l** 1428

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**a**  $2x^2 + 13x + 20$       **b**  $3x^2 + 35x + 88$       **c**  $4x^2 + 42x + 20$   
**d**  $x^2 - 6x + 9$       **e**  $2x^2 + 5x - 12$       **f**  $6x^2 + 3x - 18$

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**a** 87768      **b** 73926      **c** 53235      **d** 23532      **e** 285825

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**a** 25    **b** 4    **c** 36    **d** 12    **e** 73    **f** 16    **g** 49

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**a** 961    **b** 196    **c** 1681    **d** 676    **e** 4356    **f** 6561    **g** 3136  
**h**  $9x^2 + 24x + 16$     **i**  $25y^2 + 20y + 4$     **j**  $4x^2 - 4x + 1$     **k**  $x^2 + 14x + 49$

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**a** 186624      **b** 97969      **c** 522729      **d**  $4x^2 + 12xy + 20x + 9y^2 + 30y$  25

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**a** 8836      **b** 9604      **c** 7744      **d** 11236      **e** 10609  
**f** 992/016    **g** 976/144    **h** 1024/144    **i** 1002/001    **j** 10196/9604

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**a** 163      **b** 135      **c** 137      **d** 1465      **e** 1463

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**a** 81    **b** 644    **c** 156    **d** 2568    **e** 768

**f** 1472      **g** 2232      **h** 2332      **i** 2772      **j** 748      **k** 12176

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**a** 6561      **b** 8281      **c** 3969      **d** 3136      **e** 388129      **f** 54756

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**a** 44      **b** 72      **c** 94      **d** 67

**e** 63      **f** 56      **g** 71      **h** 66

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**a** 28      **b** 49      **c** 29

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**a** 63      **b** 53      **c** 6.5      **d** 5.7

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**a** 2 r 17      **b** 4 r 18      **c** 3 r 5      **d** 7 r 0

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**a** 321 r 0      **b** 467 r 7      **c** 1461 r 11      **d** 50 r 2

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**a** 219 r 2      **b** 104 r 32      **c** 48 r 10      **d** 3 r 13

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**a** 552.630      **b** 4585.506      **c** 14.557      **d** 0.236

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**a** 6.35      **b** 16.53      **c** 5.08      **d** 0.16

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**a**  $\frac{11}{19} = 0.\dot{5}_1 7_1 8_1 9_1 4_1 7_1 3_1 6_1 8_1 4_1 2_1 0_1 5_1 2_1 6_1 3_1 \dot{1}$

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**a**  $\frac{17}{29} = 0.\dot{5}_2 8_2 6_2 2_2 0_2 6_2 8_2 9_2 6_2 5_2 1_2 7_2 2_2 4_2 1_2 3_2 7_2 9_2 3_2 1_2 0_2 3_2 4_2 4_2 8_2 2_2 \dot{7}$

**b**  $\frac{11}{19} = 0.\dot{5}17\dot{8}9147\dot{3}168/42105263\dot{1}$

**c**  $29 - 17 = 12$  and  $\dot{1}2$  appears after 14 figures

**d**  $\frac{27}{39} = 0.\dot{3}69\dot{2}/307$

**e**  $\frac{3}{49} = 0.\dot{3}06\dot{1}2244\dot{4}897945918\dot{3}367346$   
938775510204081632653

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**a**  $\frac{29}{39} = 0.\dot{7}14\dot{3}58\dot{9}$

**b**  $\frac{19}{39} = 0.\dot{3}487\dot{1}7\dot{9}$

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**a**  $\frac{1}{7} = \frac{7}{49} = 0.\dot{2}14\dot{2}/857$

**b**  $\frac{2}{13} = \frac{6}{39} = 0.\dot{2}15\dot{3}/846$

**c**  $\frac{5}{23} = \frac{15}{69} = 0.\dot{2}17\dot{3}9\dot{1}30\dot{4}354/78260869565$

**d**  $\frac{7}{33} = \frac{21}{99} = 0.\dot{2}1$

**e**  $\frac{9}{11} = \frac{81}{99} = 0.\dot{8}1$

**f**  $\frac{3}{17} = \frac{21}{119} = 0.\dot{9}17\dot{6}47\dot{0}5\dot{8}/82352941$

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**a** 5      **b** 6      **c** 31      **d** 8      **e**  $5\frac{1}{2}$

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**a** 5      **b** 4      **c** 5      **d** 1      **e** 2      **f** 5

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**a** 1, 5      **b** 2, -6      **c** -3, -5

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**a**  $x = 0, y = 3$       **b**  $y = 0, x = 2$       **c**  $y = 0, x = 5$

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**a**  $-\frac{8}{3}$       **b**  $-\frac{3}{2}$       **c**  $\frac{1}{2}, 1$